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COBE-DMR-NORMALIZED OPEN CDM COSMOGONIES

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ABSTRACT

Cut-sky orthogonal mode analyses of the *COBE*-DMR 53 and 90 GHz sky maps are used to determine the normalization of a variety of open cosmogonical models based on the cold dark matter scenario. To constrain the allowed cosmological-parameter range for these open cosmogonies, the predictions of the DMR-normalized models are compared to various observational measures of cosmography and large-scale structure, viz.: the age of the universe; small-scale dynamical estimates of the clustered-mass density parameter Ω_0 ; constraints on the Hubble parameter h , the X-ray cluster baryonic-mass fraction Ω_B/Ω_0 , and

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the matter power spectrum shape parameter; estimates of the mass perturbation amplitude; and constraints on the large-scale peculiar velocity field.

The open-bubble inflation model (Ratra & Peebles 1994; Bucher, Goldhaber, & Turok 1995; Yamamoto, Sasaki, & Tanaka 1995) is consistent with current determinations of the 95% confidence level (c.l.) range of these observational constraints. More specifically, for a range of h , the model is reasonably consistent with recent high-redshift estimates of the deuterium abundance which suggest $\Omega_B h^2 \sim 0.007$, provided $\Omega_0 \sim 0.35$; recent high-redshift estimates of the deuterium abundance which suggest $\Omega_B h^2 \sim 0.02$ favour $\Omega_0 \sim 0.5$, while the old nucleosynthesis value $\Omega_B h^2 = 0.0125$ requires $\Omega_0 \sim 0.4$.

Small shifts in the inferred *COBE*-DMR normalization amplitudes due to: (1) the small differences between the galactic- and ecliptic-coordinate sky maps, (2) the inclusion or exclusion of the quadrupole moment in the analysis, (3) the faint high-latitude Galactic emission treatment, and, (4) the dependence of the theoretical cosmic microwave background anisotropy angular spectral shape on the value of h and Ω_B , are explicitly quantified.

The DMR data alone do not possess sufficient discriminative power to prefer any values for Ω_0 , h , or Ω_B at the 95% c.l. for the models considered. At a lower c.l., and when the quadrupole moment is included in the analysis, the DMR data are most consistent with either $\Omega_0 \lesssim 0.1$ or $\Omega_0 \sim 0.7$ (depending on the model considered). However, when the quadrupole moment is excluded from the analysis, the DMR data are most consistent with $\Omega_0 \sim 0.35 - 0.5$ in all open models considered (with $0.1 \leq \Omega_0 \leq 1$), including the open-bubble inflation model. Earlier claims (Yamamoto & Bunn 1996; Bunn & White 1996) that the DMR data require a 95% c.l. lower bound on Ω_0 (~ 0.3) are not supported by our (complete) analysis of the four-year data: the DMR data alone cannot be used to meaningfully constrain Ω_0 .

Subject headings: cosmic microwave background — cosmology: observations — large-scale structure of the universe — galaxies: formation

1. INTRODUCTION

Quantum-mechanical fluctuations during an early epoch of inflation provide a plausible mechanism to generate the energy-density perturbations responsible for observed cosmological structure. While it has been known for quite some time that inflation is

consistent with open spatial hypersurfaces (Gott 1982; Guth & Weinberg 1983), attention was initially focussed on models in which there are a very large number of e -foldings during inflation, resulting in almost exactly flat spatial hypersurfaces for the observable part of the present universe (Guth 1981; also see Kazanas 1980; Sato 1981a,b). This was, perhaps, inevitable because of strong theoretical prejudice towards flat spatial hypersurfaces and their resulting simplicity. However, to get a very large number of e -foldings during inflation it seems necessary that the inflation model have a small dimensionless parameter (J. R. Gott, private communication 1994; Banks et al. 1995), which would require an explanation.

Attempts to reconcile these “favoured” flat spatial hypersurfaces with observational measures of a low value for the clustered-mass density parameter Ω_0 have concentrated on models in which one postulates the presence of a cosmological constant Λ (Peebles 1984). In the simplest flat- Λ model one assumes a scale-invariant (Harrison 1970; Peebles & Yu 1970; Zel'dovich 1972) primordial power spectrum for gaussian adiabatic energy-density perturbations. Such a spectrum is generated by quantum-mechanical fluctuations during an early epoch of inflation in a spatially-flat model, provided that the inflaton potential is reasonably flat (Fischler, Ratra, & Susskind 1985, and references therein)¹⁰. It has been demonstrated that these models are indeed consistent with current observational constraints (e.g., Stompor, Górska, & Banday 1995; Ostriker & Steinhardt 1995; Ratra & Sugiyama 1995; Liddle et al. 1996b; Ganga, Ratra, & Sugiyama 1996b, hereafter GRS).

An alternative, more popular of late, is to accept that the spatial hypersurfaces are not flat. In this case, the radius of curvature for the open spatial sections introduces a new length scale (in addition to the Hubble length), which requires a generalization of the usual flat-space scale-invariant spectrum (Ratra & Peebles 1994, hereafter RP94). Such a spectrum is generated by quantum-mechanical fluctuations during an epoch of inflation in an open-bubble model (RP94; Ratra & Peebles 1995, hereafter RP95; Bucher et al. 1995, hereafter BGT; Lyth & Woszczyna 1995; Yamamoto et al. 1995, hereafter YST), provided that the inflaton potential inside the bubble is reasonably flat. Such gaussian adiabatic open-bubble inflation models have also been shown to be consistent with current observational constraints (RP94; Kamionkowski et al. 1994; Górska et al. 1995, hereafter GRSB; Liddle et al. 1996a, hereafter LLRV; Ratra et al. 1995; GRS).

Inflation theory by itself is unable to predict the normalization amplitude for the energy-density perturbations. Currently, the least controversial and most robust method

¹⁰ In inflation models the small observed cosmic microwave background (CMB) anisotropy could be the consequence of the small ratio of the inflation epoch mass scale to the Planck mass (Ratra 1991, and references therein; also see Banks et al. 1995).

for the normalization of a cosmological model is to fix the amplitude of the model-predicted large-scale CMB spatial anisotropy by comparing it to the observed CMB anisotropy discovered by the *COBE*-DMR experiment (Smoot et al. 1992).

Previously, specific open cold dark matter (CDM) models have been examined in light of the *COBE*-DMR two-year results (Bennett et al. 1994). GRSB investigated the CMB anisotropy angular spectra predicted by the open-bubble inflation model (RP94), and compared large-scale structure predictions of this DMR-normalized model to observational data.¹¹ Cayón et al. (1996) performed a related analysis for the open model with a flat-space scale-invariant spectrum (Wilson 1983, hereafter W83), and Yamamoto & Bunn (1996, hereafter YB) examined the effect of additional sources of quantum fluctuations (BGT; YST) in the open-bubble inflation model.

In this paper, we study the observational predictions for a number of open CDM models. In particular, we employ the power spectrum estimation technique devised by Górski (1994) for incomplete sky coverage to normalize the open models using the *COBE*-DMR four-year data (Bennett et al. 1996). In §2 we provide an overview of open-bubble inflation cosmogonies. In §3 we detail the various DMR data sets used in the analyses here, discuss the various open models we consider, and present the DMR estimate of the CMB rms quadrupole anisotropy amplitude $Q_{\text{rms-PS}}$ as a function of Ω_0 for these open models. In §4 we detail the computation of several cosmographic and large-scale structure statistics for the DMR-normalized open models. These statistics are confronted by various current observational constraints in §5. Our results are summarized in §6.

2. OPEN-BUBBLE INFLATION MODELS

The simplest open inflation model is that in which a single open-inflation bubble nucleates in a (possibly) spatially-flat, inflating spacetime (Gott 1982; Guth & Weinberg 1983). In this model, the first epoch of inflation smooths away any preexisting spatial inhomogeneities, while simultaneously generating quantum-mechanical zero-point fluctuations. Then, in a tunnelling event, an open-inflation bubble nucleates, and for a small enough nucleation probability the observable universe lies inside a single open-inflation bubble. Fluctuations of relevance to the late-time universe can be generated via three different quantum mechanical mechanisms: (1) they can be generated in the first epoch of inflation; (2) they can be generated during the tunnelling event (thus resulting in a slightly

¹¹ Ratra et al. (1995) and GRS subsequently extended the analysis to smaller scales, comparing detailed CMB anisotropy predictions to observational data.

inhomogeneous initial hypersurface inside the bubble, or a slightly non-spherical bubble); and (3) they can be generated inside the bubble. The tunneling amplitude is largest for the most symmetrical solution (and deviations from symmetry lead to an exponential suppression), so it has usually been assumed that the nucleation process (mechanism [2]) does not lead to the generation of significant inhomogeneities. Quantum-mechanical fluctuations generated during evolution inside the bubble (RP95) are significant. Assuming that the energy-density difference between the two epochs of inflation is negligible (and so the bubble wall is not significant), one may estimate the contribution to the perturbation spectrum after bubble nucleation from quantum-mechanical fluctuations during the first epoch of inflation (BGT; YST). As discussed by Bucher & Turok (1995, hereafter BT) (also see YST; YB), the observable predictions of these simple open-bubble inflation models are almost completely insensitive to the details of the first epoch of inflation, for the observationally-viable range of Ω_0 . This is because the fluctuations generated during this epoch affect only the smallest wavenumber part of the energy-density perturbation power spectrum, which cannot contribute significantly to observable quantities because of the spatial curvature length “cutoff” in an open universe (e.g., W83; Kamionkowski & Spergel 1994; RP95). Inclusion of such fluctuations in the calculations alter the predictions for the present value of the rms linear mass fluctuations averaged over an $8h^{-1}$ Mpc sphere, $(\delta M/M)[8h^{-1} \text{ Mpc}]$, by $\sim 0.1 - 0.2\%$ (which is comparable to our computational accuracy).

Besides the open-bubble inflation model spectra, a variety of alternatives have also been considered. Predictions for the usual flat-space scale-invariant spectrum in an open model have been examined (W83; Abbott & Schaefer 1986; Gouda, Sugiyama, & Sasaki 1991; Sugiyama & Gouda 1992; Kamionkowski & Spergel 1994; Sugiyama & Silk 1994; Cayón et al. 1996). The possibility that the standard formulation of quantum mechanics is incorrect in an open universe, and that allowance must be made for non-square-integrable basis functions has been investigated (Lyth & Woszczyna 1995), and other spectra have also been considered (e.g., W83; Abbott & Schaefer 1986; Kamionkowski & Spergel 1994). These spectra, being inconsistent with either standard quantum mechanics or the length scale set by spatial curvature, are of historical interest.

More recently, the open-bubble inflation scenario has been further elaborated on. YST have considered a very specific model for the nucleation of the open bubble in a spatially-flat de Sitter spacetime, and demonstrated a possible additional contribution from a non-square-integrable basis function which depends on the form of the potential, and on the assumed form of the quantum state prior to bubble nucleation¹². However, since the

¹² If the length scale set by the mass determined by the curvature of the inflaton potential in the first epoch of inflation is significantly smaller than the Hubble length, as is expected in reasonable particle physics

non-square-integrable basis function contributes only on the very largest scales, the spatial curvature “cutoff” in an open universe makes almost all of the model predictions insensitive to this basis function, for the observationally-viable range of Ω_0 (YST; YB). For example, at $\Omega_0 \sim 0.4 - 0.5$ its effect is to change $(\delta M/M)[8h^{-1} \text{ Mpc}]$ by $\sim 0.8 - 1\%^{13}$.

An additional possible effect determined for the specific model of an open-inflation bubble nucleating in a spatially-flat de Sitter spacetime is that fluctuations of the bubble wall behave like a non-square-integrable basis function (Hamazaki et al. 1996; Garriga 1996; García-Bellido 1996; Yamamoto, Sasaki, & Tanaka 1996). While there are models in which these bubble-wall fluctuations are completely insignificant (Garriga 1996; Yamamoto et al. 1996), there is as yet no computation that accounts for both the bubble-wall fluctuations as well as those generated during the evolution inside the bubble (which are always present), so it is not yet known if bubble-wall fluctuations can give rise to an observationally significant effect. Finally, again in this very specific model, the effects of a finite bubble size at nucleation seem to alter the zero bubble size predictions only by a very small amount (Yamamoto et al. 1996; Cohn 1996).

While there is no guarantee that there is a spatially-flat de Sitter spacetime prior to bubble nucleation, these computations do illustrate the important point that the spatial curvature length “cutoff” in an open universe (e.g., RP95) does seem to ensure that what happens prior to bubble nucleation does not significantly affect the observable predictions for observationally-viable single-field open-bubble inflation models. It is indeed reassuring that accounting only for the quantum mechanical fluctuations generated during the evolution inside the bubble (RP94) seems to be essentially all that is required to make observational predictions for the single-field open-bubble inflation models. That is, the observational predictions of the open-bubble inflation scenario seem to be as robust as those for the spatially-flat inflation scenario.

3. CMB ANISOTROPY NORMALIZATION PROCEDURE

models, there is no non-square-integrable basis function in the second epoch of inflation (YST).

¹³ Hence it seems that there is as yet no need to speculate about the quantum state prior to bubble nucleation. However, more recently it has been suggested that in certain two field models (Linde & Mezhlumian 1995) the contribution of this non-square-integrable basis function might be enhanced by the ratio of the energy densities before and after bubble nucleation, and it has been suggested that if this ratio is large it would be a problem for these two field models (Sasaki & Tanaka 1996). However, this depends sensitively on the speculative properties of the pre-nucleation model and vacuum state.

3.1. Data Selection and Power Spectrum Inference

In this paper, we utilize the DMR four-year 53 and 90 GHz sky maps in both galactic and ecliptic coordinates. We thus quantify explicitly the expected small shifts in the inferred normalization amplitudes due to the small differences between the galactic- and ecliptic-coordinate maps. The maps are coadded using inverse-noise-variance weights derived in each coordinate system. The least sensitive 31 GHz maps have been omitted from the analysis, since their contribution is minimal under such a weighting scheme.

The dominant source of emission in the DMR maps is due to the Galactic plane. We are unable to model this contribution to the sky temperature to sufficient accuracy to enable its subtraction, thus we excise all pixels where the Galactic-plane signal dominates the CMB. The geometry of the cut has been determined by using the DIRBE 140 μm map as a tracer of the strongest emission, as described completely in Banday et al. (1996a). All pixels with Galactic latitude $|b| < 20^\circ$ are removed, together with regions towards Scorpius-Ophiucus and Taurus-Orion. There are 3881 surviving pixels in galactic coordinates and 3890 in ecliptic. This extended (four-year data) Galactic plane cut has provided the biggest impact on the analysis of the DMR data (see Górski et al. 1996, hereafter G96).

The extent to which residual high-latitude Galactic emission can modify our results has been quantified in two ways. Since the spatial morphology of Galactic synchrotron, free-free and dust emission seems to be well described by a steeply falling power spectrum ($\sim \ell^{-3}$ – Kogut et al. 1996a, G96), the cosmological signal is predominantly compromised on the largest angular scales. As a simple test of Galactic contamination, we perform all computations both including and excluding the observed sky quadrupole. A more detailed approach (G96) notes that a large fraction of the Galactic signal can be accounted for by using the DIRBE 140 μm sky map (Reach et al. 1995) as a template for free-free and dust emission, and the 408 MHz all-sky radio survey (Haslam et al. 1981) to describe synchrotron emission. A correlation analysis yields coupling coefficients for the two templates at each of the DMR frequencies. We have repeated our model analysis after correcting the coadded sky maps by the Galactic templates scaled by the coefficients derived in G96. In particular, we adopt those values derived under the assumption that the CMB anisotropy is well-described by an $n = 1$ power law model with normalization amplitude $Q_{\text{rms-PS}} \sim 18 \mu\text{K}^{14}$. One might make criticisms of either technique: excluding information from an analysis, in this case the

¹⁴ A more self-consistent analysis would simultaneously compute the $Q_{\text{rms-PS}}$ and coupling coefficient amplitudes. In fact, we have investigated this for a sub-sample of the models considered here in which we varied Ω_0 but fixed h and Ω_B . No statistically significant changes were found in the derived values of either $Q_{\text{rms-PS}}$ or the coupling coefficients.

quadrupole components, can obviously weaken any conclusions simply because statistical uncertainties will grow; at the same time, it is not clear whether the Galactic corrections applied are completely adequate. We believe that, given these uncertainties, our analysis is the most complete and conservative one that is possible.

The power spectrum analysis technique developed by Górski (1994) is implemented. Orthogonal basis functions for the Fourier decomposition of the sky maps are constructed which specifically include both pixelization effects and the Galactic cut. (These are linear combinations of the usual spherical harmonics with multipole $\ell \leq 30$.) The functions are coordinate system dependent. A likelihood analysis is then performed as described in Górski et al. (1994).

3.2. Theoretical Spectra of Anisotropy

We consider four open model energy-density perturbation power spectra: (1) the open-bubble inflation model spectrum, accounting only for fluctuations that are generated during the evolution inside the bubble (RP94); (2) the open-bubble inflation model spectrum, now also accounting for the fluctuations generated in the first epoch of inflation (BGT; YST); (3) the open-bubble inflation model spectrum, now also accounting for both the usual fluctuations generated in the first epoch of inflation and a contribution from a non-square-integrable basis function (YST); and, (4) an open model with a flat-space scale-invariant spectrum (W83). In all cases we have ignored the possibility of tilt or primordial gravity waves, since it is unlikely that they can have a significant effect in viable open models.

With the eigenvalue of the spatial scalar Laplacian being $-(k^2+1)$, where k ($0 < k < \infty$) is the radial coordinate spatial wavenumber, the gauge-invariant fractional energy-density perturbation power spectrum of type (1) above is

$$P(k) = A \frac{(4+k^2)^2}{k(1+k^2)} T^2(k), \quad (1)$$

where $T(k)$ is the transfer function and A is the normalization amplitude¹⁵. In the simplest example, perturbations generated in the first epoch of inflation introduce an additional

¹⁵ In the literature, the primordial part of this open-bubble inflation model spectrum is occasionally called the “conformal” spectrum or the “scale-invariant” spectrum. These names are misleading: the open de Sitter spacetime inside the bubble is not conformal to spatially-flat Minkowski spacetime (more precisely, it is conformal to the upper “Milne wedge” of Minkowski spacetime), which is why the primordial part of the spectrum of eq. (1) is manifestly non-scale-invariant. This spectrum is, however, the “natural” generalization

multiplicative factor, $\coth(\pi k)$, on the right hand side of eq. (1). For a discussion of the effects of the non-square-integrable basis function see YST and YB. The energy-density power spectrum of type (4) above is

$$P(k) = AkT^2(k), \quad (2)$$

and in this case one can also consider, e.g., $P(k) \propto \sqrt{1+k^2}$ (W83), but because of the spatial curvature “cutoff” in an open model the predictions are essentially indistinguishable¹⁶. At small k the asymptotic expressions are $P(k) \propto k^{-1}$ (type 1), $P(k) \propto k^{-2}$ (type 2), and $P(k) \propto k$ (type 4).

Conventionally, the CMB fractional temperature perturbation, $\delta T/T$, is expressed as a function of angular position, (θ, ϕ) , on the sky via the spherical harmonic decomposition,

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi). \quad (3)$$

The CMB spatial anisotropy in a gaussian model can then be characterized by the angular perturbation spectrum C_{ℓ} , defined in terms of the ensemble average,

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_{\ell} \delta_{\ell \ell'} \delta_{mm'}. \quad (4)$$

The C_{ℓ} ’s used here were computed using two independent Boltzmann transfer codes developed by NS (e.g., Sugiyama 1995) and RS (e.g., Stompor 1994). Some illustrative comparisons are shown in Fig. 1. We emphasize that the excellent agreement between the C_{ℓ} ’s computed using the two codes is mostly a reflection of the currently achievable numerical accuracy. Currently, the major likely additional, unaccounted for, source of uncertainty is that due to the uncertainty in the modelling of various physical effects. The computations here assume a standard recombination thermal history, and ignore the

of the flat-space scale-invariant spectrum to the open model, and it is the open-bubble inflation model spectrum accounting only for those fluctuations generated during the evolution inside the bubble. Note that Bunn & White (1996, hereafter BW, eq. [30]) generalize the primordial part of the spectrum of eq. (1) by multiplying it with $(k^2 + 1)^{(n-1)/2}$. As yet, only the specific $n = 1$ generalized spectrum (i.e., eq. [1]) is known to be a prediction of an open-bubble inflation model and therefore consistent with the presence of spatial curvature. It is premature to draw conclusions about open cosmogony on the basis of the $n \neq 1$ version of the spectrum considered by BW.

¹⁶ It should be noted that such open model spectra are “unnatural” — they do not account properly for the additional length scale set by the radius of space curvature in an open universe. We include the case of eq. (2) here both for historical reasons and to provide a “strawman” to compare to the open-bubble inflation model.

possibility of early reionization. The simplest open models (with the least possible number of free parameters) have yet to be ruled out by observational data (GRSB; Ratra et al. 1995; GRS; this paper), so there is insufficient motivation to expand the model-parameter space by including the effect of early reionization, tilt or gravity waves¹⁷.

For the $P(k)$ of types (1), (2), and (4) above, we have evaluated the CMB anisotropy angular spectra for a range of Ω_0 spanning the interval between 0.1 and 1.0, for a variety of values of h (the Hubble parameter $H_0 = 100h \text{ km s}^{-1} \text{Mpc}^{-1}$) and the baryonic-mass density parameter Ω_B . The values of h were selected to cover the lower part of the range of ages consistent with current requirements ($t_0 \simeq 10.5$ Gyr, 12 Gyr, or 13.5 Gyr, with h as a function of Ω_0 computed accordingly; see, for example, Jimenez et al. 1996; Chaboyer et al. 1996). The values of Ω_B were chosen to be consistent with current standard nucleosynthesis requirements ($\Omega_B h^2 = 0.0055$, 0.0125, or 0.0205; e.g., Copi, Schramm, & Turner 1995; Sarkar 1996). To render the problem tractable, C_ℓ 's were determined for the central values of t_0 and $\Omega_B h^2$, and for the two combinations of these parameters which most perturb the C_ℓ 's from those computed at the central values (i.e., for the smallest t_0 we used the smallest $\Omega_B h^2$, and for the largest t_0 we used the largest $\Omega_B h^2$). Specific parameter values are given in columns (1) and (2) of Tables 1–6, and representative anisotropy spectra can be seen in Figs. 2 and 3. We therefore improve on our earlier analysis of the DMR two-year data (GRSB) by considering a suitably broader range in the (Ω_B, h) parameter space.

The CMB anisotropy spectra for $P(k)$ of type (3) above were computed for a range of Ω_0 spanning the interval between 0.1 and 0.9, for $h = 0.6$ and $\Omega_B = 0.035$. Specific parameter values are given in columns (1) and (2) of Table 7, and these spectra are shown in Fig. 4. In Fig. 5 we compare the various spectra considered here.

The differences in the low- ℓ shapes of the C_ℓ 's in the various models (Figs. 2–5) are a consequence of three effects: (1) the shape of the energy-density perturbation power spectrum at low wavenumber; (2) the exponential suppression at the spatial curvature scale in an open model; and (3) the interplay between the “usual” (fiducial CDM) Sachs-Wolfe term and the “integrated” Sachs-Wolfe (hereafter SW) term in the expression for the CMB spatial anisotropy. The relative importance of these effects is determined by the value of Ω_0 , and leads to the non-monotonic behaviour of the large-scale C_ℓ 's as a function of Ω_0 seen in

¹⁷ Note that the geometrical effect in an open universe moves the effects of early reionization on the CMB anisotropy to a smaller angular scale. As a result $Q_{\text{rms-PS}}$ values determined from the DMR data here (assuming no early reionization) are unlikely to be very significantly affected by early reionization. However, since structure forms earlier in an open model, other effects of early reionization might be more significant in an open model. While it is possible to heuristically account for such effects, an accurate quantitative estimate must await a better understanding of structure formation.

Figs. 2–5. More precisely, the contributions to the CMB anisotropy angular spectrum from the “usual” and “integrated” SW terms have a different ℓ -dependence as well as a relative amplitude that is both Ω_0 and $P(k)$ dependent.

On very large angular scales (small ℓ ’s), the dominant contribution to the “usual” SW term comes from a higher redshift (when the length scales are smaller) than does the dominant contribution to the “integrated” SW term (Hu & Sugiyama 1994, 1995). As a result, in an open model on very large angular scales, the “usual” SW term is cut off more sharply by the spatial curvature length scale than is the “integrated” SW term (Hu & Sugiyama 1994), i.e., on very large angular scales in an open model the “usual” SW term has a larger (positive) effective index n than the “integrated” SW term. On slightly smaller angular scales the “integrated” SW term is damped (i.e., it has a negative effective index n) while the “usual” SW term plateaus (Hu & Sugiyama 1994). As a consequence, going from the largest to slightly smaller angular scales, the “usual” term rises steeply and then flattens, while the “integrated” term rises less steeply and then drops (i.e., it has a peak). The change in shape, as a function of ℓ , of these two terms is both Ω_0 and $P(k)$ dependent. These are the two dominant effects at $\ell \lesssim 15 - 20$; at higher ℓ other effects come into play.

More specifically, for $\Omega_0 > 0.8$ the curvature length scale cutoff and the precise large-scale form of the $P(k)$ considered here are relatively unimportant — the CMB anisotropy angular spectrum is quite similar to that for $\Omega_0 = 1$, and the dominant contribution is the “usual” SW term. For a $P(k)$ that does not diverge at low wavenumber, as with the flat-space scale-invariant spectrum in an open model, for $\Omega_0 \lesssim 0.8$ the exponential “cutoff” at the spatial curvature length dominates, and the lowest- ℓ C_ℓ ’s are suppressed (Figs. 3 and 5). For this $P(k)$, as Ω_0 is reduced, the “usual” term continues to be important on the largest angular scales down to $\Omega_0 \sim 0.4 - 0.5$. As Ω_0 is reduced below $\sim 0.4 - 0.5$ the “integrated” term starts to dominate on the largest angular scales, and as Ω_0 is further reduced the “integrated” term also starts to dominate on smaller angular scales. From Fig. 3(a) one will notice that the “integrated” SW term “peak” first makes an appearance at $\Omega_0 = 0.4$ — the central line in the plot at $\ell(\ell + 1)C_\ell + \text{offset} \sim 3$ — and that as Ω_0 is further reduced (in descending order along the curves shown) the “integrated” term “peak” moves to smaller angular scales. The $\Omega_0 \sim 0.4$ case is where the “integrated” term peaks at $\ell \sim 2 - 3$, and the damping of this term on smaller angular scales ($\ell \gtrsim 5$) is compensated for by the steep rise of the “usual” SW term — the two terms are of roughly equal magnitude at $\ell \sim 10$ — and these effects result in the almost exactly scale-invariant spectrum at $\Omega_0 \sim 0.4$ (this case is more scale-invariant than fiducial CDM). A discussion of some of these features of the CMB anisotropy angular spectrum in the flat-space scale-invariant spectrum open model is given in Cayón et al. (1996).

Open-bubble inflation models have a $P(k)$ that diverges at low wavenumber (RP95; note that no physical quantity diverges), and this increases the low- ℓ C_ℓ 's (Figs. 2 and 5) relative to those of the flat-space scale-invariant spectrum open model (Figs. 3 and 5). The C_ℓ 's for low Ω_0 models increase more than the higher Ω_0 ones, since, for a fixed wavenumber-dependence of $P(k)$, the divergence is more prominent at lower Ω_0 (RP94). The non-square-integrable basis function (YST) contributes even more power on large angular scales, and so, at low- ℓ , the C_ℓ 's of Fig. 4 are slightly larger than those of Fig. 2 (also see Fig. 5). Again, spectra at lower values of Ω_0 are more significantly influenced.

As is clear from Figs. 2 and 5, in an open-bubble inflation model, quantum-mechanical zero-point fluctuations generated in the first epoch of inflation scarcely affect the C_ℓ 's, although at the very lowest values of Ω_0 the very lowest order C_ℓ coefficients are slightly modified. The effect is concentrated in this region of the parameter space since the fluctuations in the first inflation epoch only contribute to, and increase, the lowest wavenumber part of $P(k)$. In simple open-bubble inflation models, the precise value of this small effect is dependent on the model assumed for the first epoch of inflation (BT). Since the DMR data is most sensitive to multipole moments with $l \sim 8-10$, one expects the effect at $l \sim 2-3$ to be almost completely negligible (BT; also see YST; YB).

Figs. 3–5 show that both the flat-space scale-invariant spectrum open model, and the contribution from the non-square-integrable mode, do lead to significantly different C_ℓ 's (compared to those of Fig. 2).

3.3. Results of $Q_{\text{rms-PS}}$ fitting

The results of the DMR likelihood analyses are summarized in Figs. 6–21 and Tables 1–7 and 13.

Two representative sets of likelihood functions $L(Q_{\text{rms-PS}}, \Omega_0)$ are shown in Figs. 6 and 7. Figure 6 shows those derived from the ecliptic-frame sky maps, ignoring the correction for faint high-latitude foreground Galactic emission, and excluding the quadrupole moment from the analysis. Figure 7 shows the likelihood functions derived from the galactic-frame sky maps, accounting for the faint high-latitude foreground Galactic emission correction, and including the quadrupole moment in the analysis. Together, these two data sets span the maximum range of normalizations inferred from our analysis (the former providing the highest, and the latter the lowest $Q_{\text{rms-PS}}$).

Tables 1–7 give the $Q_{\text{rms-PS}}$ central values and $1-\sigma$ and $2-\sigma$ ranges for spectra of type (1), (3), and (4) above, computed from the appropriate posterior probability density

distribution function assuming a uniform prior. Each line in Tables 1–7 lists these values at a given Ω_0 for the 8 possible combinations of: (1) galactic- or ecliptic-coordinate map; (2) faint high-latitude Galactic foreground emission correction accounted for or ignored; and, (3) quadrupole included ($\ell_{\min} = 2$) or excluded ($\ell_{\min} = 3$)¹⁸. The corresponding ridge lines of maximum likelihood $Q_{\text{rms-PS}}$ value as a function of Ω_0 are shown in Figs. 8–10 for some of the cosmological-parameter values considered here.

Although we have computed these values for spectra of type (2) above (i.e., those accounting for perturbations generated in the first epoch of inflation) we record only a subset of them in column (4) of Table 13. These should be compared to columns (2) and (6) of Table 13, which show the maximal 2- σ $Q_{\text{rms-PS}}$ range for spectra of types (1) and (3). While the differences in $Q_{\text{rms-PS}}$ between spectra (1) and (2) [cols. (2) and (4) of Table 13] are not totally insignificant, more importantly the differences between the $(\delta M/M)[8h^{-1} \text{ Mpc}]$ values for the three spectra [cols. (3), (5), and (7) of Table 13] are observationally insignificant.

The entries in Tables 1–6 illustrate the shift in the inferred normalization amplitudes due to changes in h and Ω_B . These shifts are larger for models with a larger Ω_0 , since these models have CMB anisotropy spectra that rise somewhat more rapidly towards large ℓ , so in these cases the DMR data is sensitive to somewhat smaller angular scales where the effects of varying h and $\Omega_B h^2$ are more prominent. Figure 11 shows the effects that varying t_0 and $\Omega_B h^2$ have on some of the ridge lines of maximum likelihood $Q_{\text{rms-PS}}$ as a function of Ω_0 , and Fig. 13 illustrates the effects on some of the conditional (fixed Ω_0 slice) likelihood densities for $Q_{\text{rms-PS}}$. On the whole, for the CMB anisotropy spectra considered here, shifts in h and $\Omega_B h^2$ have only a small effect on the inferred normalization amplitude.

The normalization amplitude is somewhat more sensitive to the differences between the galactic- and ecliptic-coordinate sky maps, to the foreground high-latitude Galactic emission treatment, and to the inclusion or exclusion of the $\ell = 2$ moment. See Figs. 8–10. For the purpose of normalizing models, we choose for our 2- σ c.l. bounds values from the likelihood fits that span the maximal range in the $Q_{\text{rms-PS}}$ normalizations. Specifically, for the lower 2- σ bound we adopt the value determined from the analysis of the galactic-coordinate

¹⁸ BW have recently considered the DMR four-year data in the context of the open-bubble inflation model accounting only for the fluctuations generated during the evolution inside the bubble (RP94). However, they use an analytic approximation to the CMB anisotropy angular spectra, only consider the ecliptic-frame maps, ignore the correction for faint high-latitude Galactic foreground emission, and choose not to examine the consequences of exclusion of the quadrupole from the analysis. They also do not seem to have examined the effect on the derived $Q_{\text{rms-PS}}$ value of varying cosmological parameters like Ω_B . Since they do not quote derived $Q_{\text{rms-PS}}$ values for this model we are not able to compare to their results.

maps accounting for the high-latitude Galactic emission correction and including the $\ell = 2$ moment in the analysis, and for the upper 2σ value that determined from the analysis of the ecliptic-coordinate maps ignoring the Galactic emission correction and excluding the $\ell = 2$ moment from the analysis. These values are recorded in columns (5) and (8) of Tables 9–12, and columns (2), (4), and (6) of Table 13¹⁹.

Figure 12 compares the ridge lines of maximum likelihood $Q_{\text{rms-PS}}$ value, as a function of Ω_0 , for the four different CMB anisotropy angular spectra considered here, and Fig. 14 compares some of the conditional (fixed Ω_0 slice) likelihood densities for $Q_{\text{rms-PS}}$ for these four CMB anisotropy angular spectra.

Approximate fitting formulae may be derived to describe the above two extreme 2σ limits. For the open-bubble inflation model (RP94; BGT; YST), not including a contribution from a non-square-integrable basis function, we have

$$Q_{\text{rms-PS}}(\Omega_0)/\mu\text{K} \simeq 19_{-3.25}^{+3.50} + (4.95_{-1.2}^{+1.1} - \Omega_0) \sin[2\pi\{1 + 0.25(1.1 - \Omega_0)\}\{\Omega_0 - 0.05\}], \quad (5)$$

which is good to better than $\sim 5\%$ for all values of Ω_0 (and to better than $\sim 2\%$ over the observationally-viable range of $0.3 \lesssim \Omega \lesssim 0.6$). For those models including a contribution from the non-square-integrable basis function (YST), we have

$$Q_{\text{rms-PS}}(\Omega_0)/\mu\text{K} \simeq 21_{-4.0}^{+3.7} + (5.55_{-1.1}^{+1.1} - \Omega_0) \cos[1.25\pi\{1 + 0.25(1.1 - \Omega_0)\}\{\Omega_0 - 0.05\}], \quad (6)$$

mostly good to better than $\sim 2\%$. The flat-space scale-invariant spectrum open model fitting formula is

$$Q_{\text{rms-PS}}(\Omega_0)/\mu\text{K} \simeq 15_{-2.50}^{+2.95} + (3.25_{-0.8}^{+0.6}) \sin[2\pi(1 + 0.25\Omega_0)(\Omega_0 + 0.05) - 1.25], \quad (7)$$

generally good to better than $\sim 4\%$, except near $\Omega_0 \sim 0.1$ and $\Omega_0 \sim 1$ where the deviations are larger. Further details about these fitting formulae may be found in Stompor (1996).

The approximate fitting formulae (5)–(7) provide a convenient, portable normalization of the open models. It is important, however, to note that they have been derived using the $Q_{\text{rms-PS}}$ values determined for a given h and Ω_B , and hence do not account for the additional uncertainty (which could be as large as $\sim 2\%$) due to allowed variations in these parameters. We emphasize that in our analysis here we make use of the actual $Q_{\text{rms-PS}}$ values derived from the likelihood analyses, not these fitting formulae.

¹⁹ Since different grids (in $Q_{\text{rms-PS}}$) were used in the likelihood analyses of the various model spectra, and different interpolation methods were used in the determination of the $Q_{\text{rms-PS}}$ values, there are small (but insignificant) differences in the quoted $Q_{\text{rms-PS}}$ values for some identical models in these tables.

Figures 15 and 16 show projected likelihood densities for Ω_0 , for some of the models and DMR data sets considered here. Note that the general features of the projected likelihood densities for the open-bubble inflation model only accounting for the fluctuations generated during the evolution inside the bubble (spectrum [1] above), are consistent with those derived from the DMR two-year data (GRSB, Fig. 3). However, since we only compute down to $\Omega_0 = 0.1$ here, only the rise to the prominent peak at very low Ω_0 (GRSB) is seen. BW show in the middle left-hand panel of their Fig. 11 (presumably) the projected likelihood density for Ω_0 for the same open-bubble inflation model, the general features of which are consistent with those derived here.

Figures 17–21 show marginal likelihood densities for Ω_0 , for some of the models and DMR data sets considered here. For the open-bubble inflation model accounting only for the fluctuations generated during the evolution inside the bubble (RP94), the DMR two-year data galactic-frame (quadrupole moment excluded and included) marginal likelihoods are shown in Fig. 3 of GRSB, and are in general concord with those shown in Fig. 17 here (although, again, only the rise to the prominent low- Ω_0 peak is seen here). Note that now, especially for the quadrupole excluded case, the peaks and troughs are more prominent (although still not greatly statistically significant). Furthermore, comparing the solid line of Fig. 17(b) here to the heavy dotted line of Fig. 3 of GRSB, one notices that the intermediate Ω_0 peak is now at $\Omega_0 \lesssim 0.4$, instead of at $\Omega_0 \sim 0.5$ for the DMR two-year data. (Since BW chose not to compute for the case when the quadrupole moment is excluded from the analysis, they presumably did not notice the peak at $\Omega_0 \sim 0.35 - 0.45$ in the marginalized likelihood density for the open-bubble inflation model — see Fig. 17.)

For the open-bubble inflation model now also accounting for both the fluctuations generated in the first spatially-flat epoch of inflation (BGT; YST), and those from the non-square-integrable basis function (YST), the DMR two-year data ecliptic-frame quadrupole-included marginal likelihood (shown as the solid line in Fig. 3 of YB) is in general agreement with the dot-dashed line of Fig. 19(a). However, YB did not compute for the case where the quadrupole moment was excluded from the analysis and so did not find the peak at $\Omega_0 \sim 0.4 - 0.45$ in Fig. 19.

Given the shapes of the marginal likelihoods in Figs. 17–21, it is not at all clear if it is meaningful to derive limits on Ω_0 without making use of other (prior) information. As an example, it is not at all clear what to use for the integration range in Ω_0 . Focussing on Fig. 21(a) (which is similar to the other quadrupole excluded cases), the only conclusion seems to be that $\Omega_0 \sim 0.4$ is the value most consistent with the DMR data (at least amongst those models with $0.1 \leq \Omega_0 \leq 1$ — some of the models have another peak at $\Omega_0 < 0.1$, GRSB). However, when the quadrupole moment is included in the analysis (as in Fig. 21b),

the open-bubble inflation model peaks are at $\Omega_0 \sim 0.7$ (at least in the range $0.1 \leq \Omega_0 \leq 1$, GRSB), while the flat-space scale-invariant spectrum open model peak is at $\Omega_0 \lesssim 0.1$. At the 95% c.l. no value of Ω_0 over the range considered, 0.1–1, is excluded. (The YB and BW claims of a lower limit on Ω_0 from the DMR data alone are, at the very least, premature.)

4. COMPUTATION OF LARGE-SCALE STRUCTURE STATISTICS

The $P(k)$ (e.g., eqs. [1] and [2]) were determined from a numerical integration of the linear perturbation theory equations of motion. As before, the computations were performed with two independent numerical codes. For some of the model-parameter values considered here the results of the two computations were compared and found to be in excellent agreement. Illustrative examples of the comparisons are shown in Fig. 22. Again, we emphasize that the excellent agreement is mostly a reflection of the currently available numerical accuracy, and the most likely additional, unaccounted for, source of uncertainty is that due to the uncertainty in the modelling of various physical effects.

Table 8 list the $P(k)$ normalization amplitudes A (e.g., eqs. [1] and [2]) when $Q_{\text{rms-PS}} = 10 \mu\text{K}$. Examples of the power spectra normalized to $Q_{\text{rms-PS}}$ derived from the mean of the DMR four-year data analysis extreme upper and lower 2σ limits discussed above are shown in Figs. 23. One will notice, from Fig. 23(e), the good agreement between the open-bubble inflation spectra.

When normalized to the two extreme 2σ $Q_{\text{rms-PS}}$ limits (e.g., cols. [5] and [8] of Table 10), the $P(k)$ normalization factor (eq. [1] and Table 8) for the open-bubble inflation model (RP94; BGT; YST), may be summarized by, for the lower 2σ limit,

$$\frac{h^4 A(\Omega_0)}{10^5 \text{Mpc}^4} \simeq 4.3 + 1.95 \sin [1.07\pi(\Omega_0 - 0.1)^{0.85}], \quad (8)$$

and for the upper 2σ limit,

$$\frac{h^4 A(\Omega_0)}{10^5 \text{Mpc}^4} \simeq 9.3 + 3.35 \sin [1.13\pi(\Omega_0 - 0.1)^{0.78}]. \quad (9)$$

These fits are good to $\sim 1\%$ for $0.1 \leq \Omega_0 \leq 1$. Note however that they are derived using the $Q_{\text{rms-PS}}$ values determined for given t_0 and $\Omega_B h^2$ and hence do not account for the additional uncertainty introduced by allowed variations in these parameters (which could affect the power spectrum normalization amplitude by as much as $\sim 3 - 4\%$). From Fig. 23(e), and given the uncertainties, we see that the fitting formulae of eqs. (8) and (9) provide an adequate summary for all the open-bubble inflation model spectra.

The extreme $\pm 2\sigma$ $P(k)$ normalization factor (eq. [2] and Table 8) for the flat-space scale-invariant spectrum open model (W83) may be summarized by, for the lower 2σ limit,

$$\frac{h^4 A(\Omega_0)}{10^5 \text{Mpc}^4} \simeq 4.85 + 2.9 \cos [0.9\pi |\Omega_0 - 0.325|^{1.25}], \quad (10)$$

and for the upper 2σ limit,

$$\frac{h^4 A(\Omega_0)}{10^5 \text{Mpc}^4} \simeq 11 + 5 \cos [0.85\pi |\Omega_0 - 0.2|^{1.2}]. \quad (11)$$

These fits are good to better than $\sim 2\%$ for $0.2 \leq \Omega_0 \leq 1$; again, they are derived from $Q_{\text{rms-PS}}$ values determined at given t_0 and $\Omega_B h^2$.

Given the uncertainties involved in the normalization procedure (born of both statistical and other arguments) it is not yet possible to quote a unique DMR normalization amplitude (G96). As a “central” value for the $P(k)$ normalization factor, we currently advocate the mean of eqs. (8) and (9) or eqs. (10) and (11) as required. We emphasize, however, that it is incorrect to draw conclusions about model viability based solely on this “central” value.

In conjunction with numerically determined transfer functions, the fits of eqs. (8)–(11) allow for a determination of $(\delta M/M)[8h^{-1} \text{Mpc}]$, accurate to a few percent. Here the mean square linear mass fluctuation averaged over a sphere of coordinate radius $\bar{\chi}$ is

$$\begin{aligned} \left\langle \left[\frac{\delta M}{M}(\bar{\chi}) \right]^2 \right\rangle &= \frac{2}{\pi^2 [\sinh(\bar{\chi}) \cosh(\bar{\chi}) - \bar{\chi}]^2} \\ &\times \int_0^\infty \frac{dk}{(1+k^2)^2} [\cosh(\bar{\chi}) \sin(k\bar{\chi}) - k \sinh(\bar{\chi}) \cos(k\bar{\chi})]^2 P(k), \end{aligned} \quad (12)$$

which, on small scales, reduces to the usual flat-space expression $[9/2\pi^2] \int_0^\infty dk k^2 P(k) [\sin(k\bar{\chi}) - k\bar{\chi} \cos(k\bar{\chi})]^2 / (k\bar{\chi})^6$.

If instead use is made of the Bardeen et al. (1986, hereafter BBKS) analytic fit to the transfer function using the parameterization of eq. (13) below (Sugiyama 1995) and numerically determined values for A , the resultant $(\delta M/M)[8h^{-1} \text{Mpc}]$ values are accurate to better than $\sim 5\%$ (except for large baryon-fraction, $\Omega_B/\Omega_0 \gtrsim 0.4$, models where the error could be as large as $\sim 7\%$). Use of the analytic fits of eqs. (8)–(11) for A (instead of the numerically determined values) slightly increases the error, while use of the BBKS transfer function fit parameterized by an earlier version of eq. (13) below, $S = \Omega_0 h \exp [-\Omega_B(1 + \Omega_0)/\Omega_0]$, results in $(\delta M/M)[8h^{-1} \text{Mpc}]$ values that could be off by as much as $\sim 7 - 10\%$. Nevertheless, as has been demonstrated by LLRV, the approximate analytic fit to the transfer function greatly simplifies the computation and allows for rapid demarcation of the favoured part of cosmological-parameter space.

Numerical values for some cosmographic and large-scale structure statistics for the models considered here are recorded in Tables 9–15. We emphasize that when comparing to observational data we make use of numerically-determined large-scale structure predictions, not those derived using an approximate analytic fitting formula.

Tables 9–12 give the predictions for the open-bubble inflation model accounting only for the perturbations generated during the evolution inside the bubble (RP94), and for the flat-space scale-invariant spectrum open model (W83). Each of these tables corresponds to a different pair of $(t_0, \Omega_B h^2)$ values. The first two columns in these tables record Ω_0 and h , and the third column is the cosmological baryonic-matter fraction Ω_B/Ω_0 . The fourth column gives the value of the matter power spectrum scaling parameter (Sugiyama 1995),

$$S = \Omega_0 h e^{-\Omega_B(\sqrt{2h}+\Omega_0)/\Omega_0}, \quad (13)$$

which is used to parameterize approximate analytic fits to the power spectra derived from numerical integration of the perturbation equations. The quantities listed in columns (1)–(4) of these tables are sensitive only to the global parameters of the cosmological model.

Columns (5) and (8) of Tables 9–12 give the DMR data 2σ range of $Q_{\text{rms-PS}}$ that is used to normalize the perturbations in the models considered here. The numerical values in Table 12 are for $t_0 \simeq 12$ Gyr, $\Omega_B h^2 = 0.007$. We did not analyze the DMR data using C_ℓ 's for these models, and in this case the perturbations are normalized to the $Q_{\text{rms-PS}}$ values from the $t_0 \simeq 10.5$ Gyr, $\Omega_B h^2 = 0.0055$ analyses. (As discussed above, shifts in h and $\Omega_B h^2$ do not greatly alter the inferred normalization amplitude.)

Columns (6) and (9) of Tables 9–12 give the 2σ range of $(\delta M/M)[8h^{-1} \text{ Mpc}]$. These were determined using the $P(k)$ derived from numerical integration of the perturbation equations. For about two dozen cases, these rms mass fluctuations determined using the two independent numerical integration codes were compared and found to be in excellent agreement. (At fixed $Q_{\text{rms-PS}}$, they differ by $\sim 0.002 - 0.5\%$ depending on model-parameter values, with the typical difference being $\sim 0.1\%$. We again emphasize that this is mostly a reflection of currently achievable numerical accuracy.).

To usually better than $\sim 3\%$ accuracy, for $0.2 \leq \Omega_0 \leq 1$, the 2σ $(\delta M/M)[8h^{-1} \text{ Mpc}]$ entries of columns (6) and (9) of Tables 9–12 may be summarized by the fitting formulae listed in Table 14. These fitting formulae are more accurate than expressions for $(\delta M/M)[8h^{-1} \text{ Mpc}]$ derived at the same cosmological-parameter values using an analytic approximation to the transfer function and the normalization of eqs. (8)–(11).

For open models, as discussed below, it proves most convenient to characterize the

peculiar velocity perturbation by the parameter

$$\beta_I = \frac{\Omega_0^{0.6}}{b_{IRAS}} = 1.3\Omega_0^{0.6} \frac{\delta M}{M} (8h^{-1} \text{ Mpc}), \quad (14)$$

where b_{IRAS} is the linear bias factor for *IRAS* galaxies (e.g., Peacock & Dodds 1994). The 2σ range of β_I are listed in columns (7) and (10) of Tables 9–12.

Table 13 compares the $(\delta M/M)[8h^{-1} \text{ Mpc}]$ values for spectra of types (1)–(3) above. Clearly, there is no significant observational difference between the predictions for the different spectra. In what follows, for the open-bubble inflation model we concentrate on the type (1) spectrum above.

Again, the ranges in Tables 9–14 are those determined from the maximal 2σ $Q_{\text{rms-PS}}$ range. Table 15 lists “central DMR-normalized” values for $(\delta M/M)[8h^{-1} \text{ Mpc}]$, defined as the mean of the maximal $\pm 2\sigma$ entries of Tables 9–12. (The mean of the $\pm 2\sigma$ fitting formulae of Table 14 may be used to interpolate between the entries of Table 15.) We again emphasize that it is incorrect to draw conclusions about model viability based solely on these “central” values — for the purpose of constraining model-parameter values by, e.g., comparing numerical simulation results to observational data one must make use of computations at a few different values of the normalization selected to span the $\pm 2\sigma$ ranges of Tables 9–12.

5. CURRENT OBSERVATIONAL CONSTRAINTS ON DMR-NORMALIZED MODELS

The DMR likelihoods do not meaningfully exclude any part of the $(\Omega_0, h, \Omega_B h^2)$ parameter space for the models considered here. In this section we combine current observational constraints on global cosmological parameters with the DMR-normalized model predictions to place constraints on the range of allowed model-parameter values. It is important to bear in mind that some measures of observational cosmology remain uncertain thus our analysis here must be viewed as tentative and subject to revision as the observational situation approaches equilibrium. To constrain our model-parameter values we have employed the most robust of the current observational constraints. Tables 9–12 list some observational predictions for the models considered here, and the boldface entries are those that are inconsistent with current observational data at the 2σ significance level.

5.1. Observational Constraints Used

For each cosmographic or large-scale parameter, we have generally chosen to use constraints from a single set of observations or from a single analysis. We generally use the most recent analyses since we assume that they incorporate a better understanding of the uncertainties, especially those due to systematics. The specific constraints we use are summarized below, where we compare them to those derived from other analyses.

The model predictions depend on the age of the universe t_0 . To reconcile the models with the high measured values of the Hubble parameter h , we have chosen to focus on $t_0 \simeq 10.5, 12$, and 13.5 Gyr, which are near the lower end of the ages now under discussion. For instance, Jimenez et al. (1996) find that the oldest globular clusters have ages $\sim 11.5 - 15.5$ Gyr (also see Salaris, Degl'Innocenti, & Weiss 1996; Renzini et al. 1996), and that it is very unlikely that the oldest clusters are younger than 9.7 Gyr.

The value of Ω_0 is another input parameter for our computations. As summarized by Peebles (1993, §20), on scales $\lesssim 10h^{-1}$ Mpc a variety of different observational measurements indicate that Ω_0 is low. For instance, virial analyses of X-ray cluster data indicates $\Omega_0 = 0.24$, with a 2σ range: $0.04 < \Omega_0 < 0.44$ (Carlberg et al. 1996 — we have added their 1σ statistical and systematic uncertainties in quadrature and doubled to get the 2σ uncertainty). In a CDM model in which structure forms at a relatively high redshift (as is observed), these local estimates of Ω_0 do constrain the global value of Ω_0 (since, in this case, it is inconceivable that the pressureless CDM is much more homogeneously distributed than is the observed baryonic mass). We hence adopt a 2σ upper limit of $\Omega_0 < 0.6$ to constrain the CDM models we consider here. (This large upper limit allows for the possibility that the models might be moderately biased.) The boldface entries in column (1) of Tables 9–12 indicates those Ω_0 values inconsistent with this constraint.

Column (2) of Tables 9–12 gives the value of the Hubble parameter h that corresponds to the chosen values of Ω_0 and t_0 . Current observational data favours a larger h (e.g., Kennicutt, Freedman, & Mould 1995; Baum et al. 1995; van den Bergh 1995; Sandage et al. 1996; Ruiz-Lapuente 1996; Riess, Press, & Kirshner 1996; but also see Schaefer 1996; Branch et al. 1996). For the purpose of our analysis here we adopt the *HST* value $h = 0.69 \pm 0.08$ (1σ uncertainty, Tanvir et al. 1995); doubling the uncertainty, the 2σ range is $0.53 \leq h \leq 0.85$. The bold face entries in column (2) of Tables 9–12 indicates those model-parameter values which predict an h inconsistent with this range.

Comparison of the standard nucleosynthesis theoretical predictions for the primordial light element abundances to what is determined by extrapolation of the observed abundances to primordial values leads to constraints on $\Omega_B h^2$. It has usually been argued

that ^4He and ^7Li allow for the most straightforward extrapolation from the locally observed abundances to the primordial values (e.g., Dar 1995; Fields & Olive 1996; Fields et al. 1996, hereafter FKOT). The observed ^4He and ^7Li abundances then suggest $\Omega_B h^2 = 0.0066$, and a conservative assessment of the uncertainties indicate a 2σ range: $0.0051 < \Omega_B h^2 < 0.016$ (FKOT; also see Copi et al. 1995; Sarkar 1996).

Observational constraints on the primordial deuterium (D) abundance should, in principle, allow for a tightening of the allowed $\Omega_B h^2$ range. There are now a number of different estimates of the primordial D abundance, and since the field is still in its infancy it is, perhaps, not surprising that the different estimates are somewhat discrepant. Songaila et al. (1994), Carswell et al. (1994), and Rutgers & Hogan (1996a,b) use observations of three high-redshift absorption clouds to argue for a high primordial D abundance and so a low $\Omega_B h^2$. Tytler, Fan, & Burles (1996) and Burles & Tytler (1996) study two absorption clouds and argue for a low primordial D abundance and so a high $\Omega_B h^2$. Carswell et al. (1996) and Wampler et al. (1996) examine other absorption clouds, but are not able to strongly constrain $\Omega_B h^2$. While the error bars on $\Omega_B h^2$ determined from these D abundance observations are somewhat asymmetric, to use these results to qualitatively pick the $\Omega_B h^2$ values we wish to examine we assume that the errors are gaussian (and where needed add all uncertainties in quadrature to get the 2σ uncertainties). The large D abundance observations suggest $\Omega_B h^2 = 0.0062$ with a 2σ range: $0.0046 < \Omega_B h^2 < 0.0078$ (Rutgers & Hogan 1996a). When these large D abundances are combined with the observed ^4He and ^7Li abundances, they indicate $\Omega_B h^2 = 0.0064$, with a 2σ range: $0.0055 < \Omega_B h^2 < 0.0087$ (FKOT). The large D abundances are consistent with the standard interpretation of the ^4He and ^7Li abundances, and with the standard model of particle physics (with three massless neutrino species); they do, however, seem to require a modification in galactic chemical evolution models to be consistent with local determinations of the D and ^3He abundances (e.g., FKOT; Cardall & Fuller 1996). The low D abundance observations favour $\Omega_B h^2 = 0.024$ with a 2σ range: $0.018 < \Omega_B h^2 < 0.030$ (Burles & Tytler 1996). The low D abundance observations seem to be more easily accommodated in modifications of the standard model of particle physics, i.e., they are difficult to reconcile with exactly three massless neutrino species; alternatively they might indicate a gross, as yet unaccounted for, uncertainty in the observed ^4He abundance (Burles & Tytler 1996; Cardall & Fuller 1996). The low D abundance is approximately consistent with locally-observed D abundances, but probably requires some modification in the usual galactic chemical evolution model for ^7Li (Burles & Tytler 1996; Cardall & Fuller 1996).

To accommodate the range of $\Omega_B h^2$ now under discussion, we compute model predictions for $\Omega_B h^2 = 0.0055$ (Table 9), 0.007 (Table 12), 0.0125 (Table 10), and 0.0205 (Table 11). We shall find that this uncertainty in $\Omega_B h^2$ precludes determination of robust

constraints on model-parameter values. Fortunately, recent improvements in observational capabilities should eventually lead to a tightening of the constraints on $\Omega_B h^2$, and so allow for tighter constraints on the other cosmological parameters.

Column (3) of Tables 9–12 give the cosmological baryonic-mass fraction for the models we consider here. The cluster baryonic-mass fraction is the sum of the cluster galactic-mass and gas-mass fractions. Assuming that the White et al. (1993) 1- σ uncertainties on the cluster total, galactic, and gas masses are gaussian and adding them in quadrature, we find for the 2- σ range of the cluster baryonic-mass fraction:

$$\frac{M_B}{M_{\text{total}}} = (1 \pm 0.55) \left(0.009 + \frac{0.05}{h^{1.5}} \right). \quad (15)$$

Elbaz, Arnaud, & Böhringer (1995), White & Fabian (1995), David, Jones, & Forman (1995), Markevitch et al. (1996), and Buote & Canizares (1996) find similar (or larger) gas-mass fractions. Note that Elbaz et al. (1995) and White & Fabian (1995) find that the gas-mass error bars are somewhat asymmetric; this non-gaussianity is ignored here. Assuming that the cluster baryonic-mass fraction is an unbiased estimate of the cosmological baryonic-mass fraction, we may use eq. (15) to constrain the cosmological parameters. The boldface entries in column (3) of Tables 9–12 indicates those model-parameter values which predict a cosmological baryonic-mass fraction inconsistent with the range of eq. (15).

Viana & Liddle (1996, hereafter VL) have reanalyzed the combined galaxy $P(k)$ data of Peacock & Dodds (1994), ignoring some of the smaller scale data where nonlinear effects might be somewhat larger than previously suspected. Using an analytic approximation to the $P(k)$, they estimate that the scaling parameter (eq. [13])²⁰ $S = 0.23$, with a 2- σ range,

$$0.20 \leq S \leq 0.27. \quad (16)$$

This estimate is consistent with earlier ones²¹. It might be of interest to determine whether the wiggles in $P(k)$ due to the pressure in the photon-baryon fluid, see Figs. 23, can significantly affect the determination of S , especially in large Ω_B/Ω_0 models. (These wiggles are not well described by the analytic approximation to $P(k)$.) The boldface entries in column (4) of Tables 9–12 indicates those model-parameter values which predict a scaling parameter value inconsistent with the range of eq. (16).

²⁰ VL actually set $2h = 1$ in the exponent of eq. (13), so the numerical values of their constraint on S should be reduced slightly. We ignore this small effect here.

²¹ LLRV used results from an earlier analysis which favoured larger values of S than eq. (16) — this is one reason why LLRV favour a higher Ω_0 for the open-bubble inflation model than do GRSB.

To determine the value of the linear bias parameter b ,

$$\frac{\delta N}{N}(8h^{-1} \text{ Mpc}) = b \frac{\delta M}{M}(8h^{-1} \text{ Mpc}), \quad (17)$$

where $\delta N/N$ is the rms fractional perturbation in galaxy number, we adopt the APM value (Maddox, Efstathiou, & Sutherland 1996) of $(\delta N/N)[8h^{-1} \text{ Mpc}] = 0.96$, with 2σ range:

$$0.75 < \frac{\delta N}{N}(8h^{-1} \text{ Mpc}) < 1.2, \quad (18)$$

where we have added the uncertainty due to the assumed cosmological model and due to the assumed evolution in quadrature with the statistical 1σ uncertainty (Maddox et al. 1996, eq. [43]), and doubled to get the 2σ uncertainty. The range of eq. (18) is consistent with that determined from eqs. (7.33) and (7.73) of Peebles (1993).

The local abundance of rich clusters, as a function of their X-ray temperature, provides a tight constraint on $(\delta M/M)[8h^{-1} \text{ Mpc}]$. Eke, Cole, & Frenk (1996, hereafter ECF) (and S. Cole, private communication 1996) find for the open model at 2σ :

$$\frac{\delta M}{M}(8h^{-1} \text{ Mpc}) = (0.52 \pm 0.08)\Omega_0^{-0.46+0.10\Omega_0}, \quad (19)$$

where we have assumed that the ECF uncertainties are gaussian²². The constraints of eq. (19) are consistent with, but more restrictive than, those derived by VL²³. This is because ECF use observational data over a larger range in X-ray temperature to constrain $\delta M/M$, and also use N-body computations at $\Omega_0 = 0.3$ and 1 to calibrate the Press-Schechter model (which is used in their determination of the constraints). Furthermore, ECF also make use of hydrodynamical simulations of a handful of individual clusters in the fiducial CDM model ($\Omega_0 = 1$) to calibrate the relation between the gas temperature and the cluster mass, and then use this calibrated relation for the computations at all values of Ω_0 . The initial conditions for all the simulations were set using the analytical approximation to $P(k)$, so again it might be of interest to see whether the wiggles in the numerically integrated $P(k)$ could significantly affect the determination of the constraints of eq. (19). Kitayama & Suto

²² Note that the constraint of eq. (19) is that derived for a fixed S , and that in general it depends weakly on the value of S (and so on the value of h and Ω_B) — see Fig. 13 of ECF. In our preliminary analysis here we ignore this mild dependence on h and Ω_B . Also note that the constraint of eq. (19) is approximately that required for consistency with the observed cluster correlation function.

²³ VL favour $(\delta M/M)[8h^{-1} \text{ Mpc}] = 0.60$ for fiducial CDM, which is at the $+2\sigma$ limit of eq. (19). (As discussed in ECF, this is because VL normalize to the cluster temperature function at 7 keV, where there is a rise in the temperature function.) This is one reason why LLRV favour a higher value of Ω_0 for the open-bubble inflation model than did GRSB.

(1996) use X-ray cluster data, and a method that allows for the fact that clusters need not have formed at the redshift at which they are observed, to directly constrain the value of Ω_0 for CDM cosmogonies normalized by the DMR two-year data. Their conclusions are in resonable accord with what would be found by using eq. (19) (derived assuming that observed clusters are at their redshifts of formation). However, Kitayama & Suto (1996) note that evolution from the redshift of formation to the redshift of observation can affect the conclusions, so a more careful comparison of these two results is warranted. The boldface entries in columns (6) and (9) of Tables 9–12 indicate those model-parameter values whose predictions are inconsistent with the constraints of eq. (19)²⁴.

From large-scale peculiar velocity observational data Zaroubi et al. (1996) estimate $(\delta M/M)[8h^{-1} \text{ Mpc}] = (0.85 \pm 0.2)\Omega_0^{-0.6}$ (2- σ). It might be significant that the large-scale peculiar velocity observational data constraint is somewhat discordant with (higher than) the cluster temperature function constraint.

Since J_3 is less sensitive to smaller length scales (compared to $(\delta M/M)[8h^{-1} \text{ Mpc}]$), observational constraints on J_3 are more reliably contrasted with the linear theory predictions. However, since J_3 is sensitive to larger length scales, the observational constraints on J_3 are significantly less restrictive than the $\pm 8\%$ (1- σ) constraints of eq. (19), and so we do not record the predicted values of J_3 here.

Observational constraints on the mass power spectrum determined from large-scale peculiar velocity observations provide another constraint on the mass fluctuations. Kolatt & Dekel (1995) find at the 1- σ level

$$h^3 P(k/h = 0.1 \text{ Mpc}^{-1}) = (4.6 \pm 2.3) \times 10^3 \Omega_0^{-1.2} \text{ Mpc}^3, \quad (20)$$

where the 1- σ uncertainty also accounts for sample variance (T. Kolatt, private communication 1996). Since the uncertainties associated with the constraint of eq. (19) are more restrictive than those associated with the constraint of eq. (20), we do not tabulate predictions for this quantity here. However, comparison may be made to the predicted linear theory mass power spectra of Figs. 23, bearing in mind the $\sim \pm 4.6$ (2- σ) uncertainty of eq. (20) (the uncertainty is approximately gaussian, T. Kolatt, private communication 1996),²⁵ and the uncertainty in the DMR normalization (not shown in Figs. 23).

²⁴ Given the $\sim \pm 8\%$ (1- σ) uncertainty of eq. (19), approximate analyses based on using the analytic BBKS approximation to the transfer function should make use of the more accurate parameterization of eq. (13) (rather than that with $2h = 1$ in the exponent), as this gives $(\delta M/M)[8h^{-1} \text{ Mpc}]$ to better than $\sim 5\%$ in the observationally viable part of parameter space (provided use is made of the numerically determined values of A).

²⁵ Thus at the higher, $\sim 2\sigma$, significance level, eq. (20) provides a strong upper limit on $P(k/h =$

Columns (7) and (10) of Tables 9–12 give the DMR-normalized model predictions for β_I (eq. [14]). Cole, Fisher, & Weinberg (1995) measure the anisotropy of the redshift space power spectrum of the *IRAS* 1.2 Jy survey and conclude $\beta_I = 0.52$ with a 2σ c.l. range:

$$0.24 \leq \beta_I \leq 0.80, \quad (21)$$

where we have doubled the error bars of eq. (5.1) of Cole et al. (1995) to get the 2σ range. Cole et al. (1995, Table 1) compare the estimate of eq. (21) to other estimates of β_I , and at 2σ all estimates of β_I are consistent. It should be noted that the model predictions of β_I (eq. [14]) in Tables 9–12 assume that for *IRAS* galaxies $(\delta N/N)[8h^{-1} \text{ Mpc}] = 1/1.3$ holds exactly, i.e., they ignore the uncertainty in the rms fractional perturbation in *IRAS* galaxy number, which is presumably of the order of that in eq. (18). As the constraints from the deduced β_I values, eq. (21), are not yet as restrictive as those from other large-scale structure measures, we do not pursue this issue in our analysis here. The boldface entries in columns (7) and (10) of Tables 9–12 indicate those model-parameter values whose predictions are inconsistent with the constraints of eq. (21).

5.2. Constraints on Model-Parameter Values

The boldface entries in Tables 9–12 summarize the current constraints imposed by the observational data discussed in the previous section on the model-parameter values for the open-bubble inflation model (spectra of type [1] above), and for the flat-space scale-invariant spectrum open model (type [4] above). The current observational constraints on the models are not dissimilar, but this is mostly a reflection of the uncertainty on the constraints themselves since the model predictions are fairly different.

In the following discussion of the preferred part of model-parameter space we focus on the open-bubble inflation model (RP94). Note from Table 13 that the large-scale structure predictions of the open-bubble inflation model do not depend on perturbations generated in the first epoch of inflation (BGT; YST), and also do not depend significantly on the contribution from the non-square-integrable basis function (YST).

Table 9 corresponds to the part of parameter space with “maximized” small-scale power in matter fluctuations. This is accomplished by picking a low $t_0 \simeq 10.5$ Gyr (and so large h), and by picking a low $\Omega_B h^2 = 0.0055$ (this is the lower 2σ limit from standard nucleosynthesis and the observed ${}^4\text{He}$, ${}^7\text{Li}$, and high D abundances, FKOT).

0.1 Mpc^{-1}), especially at larger Ω_0 because of the Ω_0 dependence.

The tightest constraints on the model-parameter values come from the matter power spectrum observational data constraints on the shape parameter S (Table 9, col. [4]), and from the cluster X-ray temperature function observational data constraints on $(\delta M/M)[8h^{-1} \text{ Mpc}]$ (col. [6]). Note that for $\Omega_0 = 0.3$ the predicted upper 2σ value of $(\delta M/M)[8h^{-1} \text{ Mpc}] = 0.69$, while ECF conclude that at 2σ the observational data requires that this be at least 0.74, so an $\Omega_0 = 0.3$ case fails this test. The constraints on β_I (col. [7]) are not as restrictive as those on $(\delta M/M)[8h^{-1} \text{ Mpc}]$. For these values of t_0 and $\Omega_B h^2$ the cosmological baryonic-mass fraction at $\Omega_0 = 0.3$ is predicted to be 0.033 (col. [3]), while at 2σ White et al. (1993) require that this be at least 0.039 (at $h = 0.75$), so again this $\Omega_0 = 0.3$ model just fails this test. Given the observational uncertainties, it might be possible to make minor adjustments to model-parameter values so that an $\Omega_0 \sim 0.3 - 0.35$ model with $t_0 \sim 10.5$ Gyr and $\Omega_B h^2 \sim 0.0055$ is just consistent with the observational data. However, it is clear that current observational data do not favour an open model with $\Omega_B h^2 \simeq 0.0055$ — the observed cluster $(\delta M/M)[8h^{-1} \text{ Mpc}]$ favours a larger Ω_0 while the observed cluster baryonic-mass fraction favours a smaller Ω_0 , and so are in conflict.

Table 10 gives the predictions for the $t_0 \simeq 12$ Gyr, $\Omega_B h^2 = 0.0125$ models. This value of $\Omega_B h^2$ is consistent with the 2σ range determined from standard nucleosynthesis and the observed ${}^4\text{He}$ and ${}^7\text{Li}$ abundances: $0.0051 < \Omega_B h^2 < 0.016$ (FKOT, also see Copi et al. 1995; Sarkar 1996). It is, however, somewhat difficult to reconcile $\Omega_B h^2 = 0.0125$ with the 2σ range derived from the observed ${}^4\text{He}$, ${}^7\text{Li}$, and current high D abundances $0.0055 < \Omega_B h^2 < 0.0087$ (FKOT), or with that from the current observed low D abundances $0.018 < \Omega_B h^2 < 0.030$ (Burles & Tytler 1996). In any case, the observed D abundances are still under discussion, and must be viewed as preliminary. In this case, open-bubble inflation models with $0.35 < \Omega_0 \lesssim 0.5$ are consistent with the observational constraints. The current central observational data values for S and β_I favour $\Omega_0 \sim 0.4$, while that for the cluster baryonic-mass fraction prefers $\Omega_0 \sim 0.3$, and that for $(\delta M/M)[8h^{-1} \text{ Mpc}]$ favours $\Omega_0 \sim 0.45$, so in this case the agreement between predictions and observational data is fairly impressive (although the Tanvir et al. 1995 central h value favours $\Omega_0 \sim 0.2$). Note that in this case models with $\Omega_0 \gtrsim 0.6$ are quite inconsistent with the data.

Table 11 gives the predictions for $t_0 \simeq 13.5$ Gyr, $\Omega_B h^2 = 0.205$ models. This baryonic-mass density value is consistent with that determined from the current observed low D abundances, but is difficult to reconcile with the current standard nucleosynthesis interpretation of the observed ${}^4\text{He}$ and ${}^7\text{Li}$ abundances (Cardall & Fuller 1996). The larger value of $\Omega_B h^2$ (and smaller value of h) has now lowered small-scale power in mass fluctuations somewhat significantly, opening up the allowed Ω_0 range to larger values. Models with $0.4 < \Omega_0 < 0.6$ are consistent with the observational data, although the higher Ω_0 part of the range is starting to conflict with what is determined from the

small-scale dynamical estimates, and the models do require a somewhat low h (but not yet inconsistently so at the 2σ significance level — while the Tanvir et al. 1995 central h value requires $\Omega_0 < 0.1$, at 2σ the h constraint only requires $\Omega_0 \lesssim 0.6$). The central observational values for S , the cluster baryonic-mass fraction, $(\delta M/M)[8h^{-1} \text{ Mpc}]$, and β_I favour $\Omega_0 \sim 0.5$, so the agreement with observational data is fairly impressive, and could even be improved by reducing t_0 a little to raise h .

Table 12 gives the predictions for another part of model-parameter space. Here we show $\Omega_B h^2 = 0.007$ models (at $t_0 \simeq 12$ Gyr), consistent with the central value of $\Omega_B h^2$ determined from standard nucleosynthesis using the observed ^4He , ^7Li , and high D abundances (FKOT). The larger value of $\Omega_B h^2$ (compared to Table 9) eases the cluster baryonic-mass fraction constraint, which now requires only $\Omega_0 < 0.4$. The increase in $\Omega_B h^2$ also decreases the mass fluctuation amplitude, making it more difficult to argue for $\Omega_0 = 0.3$; however, models with $0.35 < \Omega_0 < 0.4$ seem to be consistent with the observational constraints when $\Omega_B h^2 \sim 0.007$ and $t_0 \sim 12$ Gyr. It is interesting that in this case the central observational data values we consider for S , for $(\delta M/M)[8h^{-1} \text{ Mpc}]$, and for β_I prefer $\Omega_0 \sim 0.4$; however, that for the cluster baryonic-mass fraction (as well as that for h) favours $\Omega_0 \sim 0.2$ (although at 2σ the cluster baryonic-mass fraction constraint only requires $\Omega_0 < 0.4$). Hence, while $\Omega_0 \sim 0.35 - 0.4$ open-bubble inflation models with $\Omega_B h^2 \sim 0.007$ and $t_0 \sim 12$ Gyr are quite consistent with the observational constraints, in this case the agreement between predictions and observations is not spectacular. Note that in this case models with $\Omega_0 \gtrsim 0.5 - 0.6$ are quite inconsistent with the observational data.

In summary, open-bubble inflation models based on the CDM picture (RP94; BGT; YST) are reasonably consistent with current observational data provided $0.3 < \Omega_0 \lesssim 0.6$. The flat-space scale-invariant spectrum open model (W83) is also reasonably compatible with current observational constraints for a similar range of Ω_0 . The uncertainty in current estimates of $\Omega_B h^2$ is one of the major reasons why such a large range in Ω_0 is consistent with current observational constraints.

Our previous analysis of the DMR two-year data led us to conclude that only those open-bubble inflation models near the lower end of the above range ($\Omega_0 \sim 0.3 - 0.4$) were consistent with the majority of observations (GRSB). The increase in the allowed range to higher Ω_0 values $\sim 0.5 - 0.6$ can be ascribed to a number of small effects. Specifically, these are: (1) the slight downward shift in the central value of the DMR four-year normalization relative to the two-year one (G96); (2) use of the full 2σ range of normalizations allowed by the DMR data analysis (instead of the 1σ range allowed by the galactic-frame quadrupole-excluded DMR two-year data set used previously); (3) use of the 2σ range of the small-scale dynamical estimates of Ω_0 instead of the 1σ range used in

our earlier analysis; (4) we consider a range of $\Omega_B h^2$ values here (in GRSB we focussed on $\Omega_B h^2 = 0.0125$); and (5) we consider a range of t_0 values here (in GRSB we concentrated on $t_0 \simeq 12$ Gyr). We emphasize, however, that the part of parameter space with $\Omega_0 \sim 0.5 - 0.6$ is only favoured if $\Omega_B h^2$ is large (> 0.02), h is low (< 0.55), and the small-scale dynamical estimates of Ω_0 turn out to be biased somewhat low.

5.3. Indications from Additional Observational Constraints

The observational results we have used to constrain model-parameter values in the previous sections are the most robust currently available. In addition, there are several other observational results which we do not consider to be as robust, and any conclusions drawn from these should be treated with due caution. In this section we summarize several of the more tentative constraints from more recent observations.

In our analysis of the DMR two-year data normalized models, we compared model predictions for the rms value of the smoothed peculiar velocity field to results from the analysis of observational data (Bertschinger et al. 1990). We do not do so again here since, given the uncertainties, the conclusions drawn in GRSB are not significantly modified. In particular, comparison of the appropriate quantities implies that we can treat the old $1-\sigma$ upper limits essentially as $2-\sigma$ upper limits for the four-year analysis.

In GRSB we used β_I determined by Nusser & Davis (1994), $0.2 < \beta_I < 1.0$ ($2-\sigma$), to constrain the allowed range of models to $0.2 < \Omega_0 \lesssim 0.6$. Here we use the Cole et al. (1995) estimate, $0.24 < \beta_I < 0.80$ ($2-\sigma$), which, for the models of Table 10, requires $\Omega_0 > 0.25$. This value is just slightly below the lower limit ($\Omega_0 \gtrsim 0.3$) derived from the Bertschinger et al. (1990) results in GRSB. We hence conclude that the large-scale flow results of Bertschinger et al. (1990) indicates a lower $2-\sigma$ limit on Ω_0 that is about $\Delta\Omega_0 \sim 0.05$ higher than that suggested by the redshift-space distortion analysis of Cole et al. (1995).²⁶ We however strongly emphasize that the central value of the large-scale flow results of Bertschinger et al. (1990) does favour a significantly larger value of Ω_0 than the rest of the data we have considered here. Furthermore, as discussed in detail in GRSB, there is some uncertainty in how to properly interpret large-scale velocity data in the open models, particularly given the large sample variance associated with the measurement of a single bulk velocity (Bond 1996, also see LLRV). A more careful analysis, as well as more observational data, is undoubtedly needed before it will be possible to robustly conclude that the large-scale velocity data does

²⁶ Note that the lower limit from the Bertschinger et al. (1990) analysis is not as restrictive as that set by the cluster X-ray temperature function data constraints on $(\delta M/M)[8h^{-1} \text{ Mpc}]$.

indeed force one to consider significantly larger values of Ω_0 than is favoured by the rest of the observational constraints (and hence rules out the models considered here).

It might be significant that on comparing the mass power spectrum deduced from a refined set of peculiar velocity observations to the galaxy power spectrum determined from the APM survey, Kolatt & Dekel (1995) estimate that for the optically-selected APM galaxies $\beta = 0.80$ with a 2σ range,

$$0.60 < \beta < 1.0. \quad (22)$$

(Note that it has been argued that systematic uncertainties preclude a believable determination of β_I from a comparison of the observed large-scale peculiar velocity field to the *IRAS* 1.2 Jy galaxy distribution, Davis, Nusser, & Willick 1996.) This range is consistent with other estimates now under discussion. The Stromlo-APM comparison of Loveday et al. (1996) indicates $\beta \simeq 0.48$, with a 2σ upper limit of 0.75, while Baugh (1996) concludes that $\beta < 1.0$ (2σ), and Ratcliffe et al. (1996) argue for $\beta = 0.55 \pm 0.12$. Using the APM range for $(\delta N/N)[8h^{-1} \text{ Mpc}]$, eq. (18), the Kolatt & Dekel (1995) estimate of β , eq. (22), may be converted to an estimate of $\delta M/M$, and at 2σ ,

$$\frac{\delta M}{M}(8h^{-1} \text{ Mpc}) = (0.45 - 1.2)\Omega_0^{-0.6}. \quad (23)$$

It is interesting that at $\Omega_0 = 1$ the lower part of this range is consistent with that determined from the cluster X-ray temperature function data, eq. (19), although at lower Ω_0 eq. (23) indicates a larger value than does eq. (19) because of the steeper rise to low Ω_0 .

Zaroubi et al. (1996) have constrained model-parameter values by comparing large-scale flow observations to that predicted in the DMR two-year data normalized open-bubble inflation model. They conclude that the open-bubble inflation model provides a good description of the large-scale flow observations if, at 2σ ,

$$0.31 < \Omega_0 h < 0.44. \quad (24)$$

From Table 12 we see that an open-bubble inflation model with $\Omega_0 = 0.45$ and $h = 0.62$ provides a good fit to all the observational data considered in §5.1. For $h = 0.62$ Zaroubi et al. (1996) conclude that at 2σ $\Omega_0 > 0.5$ (eq. [24]), just above our value of $\Omega_0 = 0.45$. Since the Zaroubi et al. (1996) analysis does not account for the uncertainty in the DMR normalization (T. Kolatt, private communication 1996), it is still unclear if the constraints from the large-scale flow observations are in conflict with those determined from the other data considered here (and so rule out the open-bubble inflation model). It might also be significant that on somewhat smaller length scales there is support for a smaller value of Ω_0 from large-scale velocity field data (Shaya, Peebles, & Tully 1995).

The cluster peculiar velocity function provides an alternate mechanism for probing the peculiar velocity field (e.g., Croft & Efstathiou 1994; Moscardini et al. 1995; Bahcall & Oh 1996). Bahcall & Oh (1996) conclude that current observational data is well-described by an $\Omega_0 = 0.3$ flat- Λ model with $h = 0.67$ and $(\delta M/M)[8h^{-1} \text{ Mpc}] = 0.67$. This normalization is somewhat smaller than that indicated by the DMR data (e.g., Ratra & Sugiyama 1995). While Bahcall & Oh (1996) did not compare the cluster peculiar velocity function data to the predictions of the open-bubble inflation model, approximate estimates indicate that this data is consistent with the open-bubble inflation model predictions for the range of Ω_0 favoured by the other data we consider in §§5.1, 5.2 — see the $(\delta M/M)[8h^{-1} \text{ Mpc}]$ values for the allowed models in Tables 9–12. Bahcall & Oh (1996) also note that it is difficult, if not impossible, to reconcile the cluster peculiar velocity observations with what is predicted in high density models like fiducial CDM and MDM.

At fixed $(\delta M/M)[8h^{-1} \text{ Mpc}]$, low-density cosmogonies form structure earlier than high density ones. Thus observations of structure at high redshift may be used to constrain the matter density. As benchmarks, we note that scaling from the results of the numerical simulations of Cen & Ostriker (1993), in a open model with $(\delta M/M)[8h^{-1} \text{ Mpc}] = 0.8$ galaxy formation peaks at a redshift $z_g \simeq 2.3$ when $\Omega_0 = 0.45$ and at $z_g \simeq 2.5$ when $\Omega_0 = 0.4$. Thus the open-bubble inflation model is not in conflict with observational indications that the giant elliptical luminosity function at $z \sim 1$ is similar to that at the present (e.g., Lilly et al. 1995; Glazebrook et al. 1995; Im et al. 1996), nor is it in conflict with observational evidence for massive galactic disks at $z \sim 1$ (Vogt et al. 1996). These models can also accommodate observational evidence of massive star-forming galaxies at $z \sim 1.5$ (Cowie, Hu, & Songaila 1995), as well as the significant peak at $z \sim 2.2$ in the number of galaxies as a function of (photometric) redshift found in the Hubble Deep Field (Gwyn & Hartwick 1996), and it is not inconceivable that objects like the $z = 2.7$ “protogalaxy” candidate²⁷ (Yee et al. 1996; Ellingson et al. 1996) can be produced in these models. It is, however, at present unclear whether the open-bubble inflation model can accommodate a substantial population of massive star-forming galaxies at $z \sim 3 - 3.5$ (Steidel et al. 1996; Giavalisco, Steidel, & Macchetto 1996), and if there are many more examples of massive damped Lyman α systems²⁸ like the one at $z = 4.4$ (e.g., Lu et al. 1996; Wampler et al. 1996;

²⁷ Note that if the velocity dispersion of the nearby foreground cluster has actually been significantly underestimated, the striking properties of this object could mostly be a consequence of gravitational lensing and it would seem to be more reasonably interpreted as a massive star-forming galaxy (Williams & Lewis 1996).

²⁸ These have many of the properties expected of young galaxies (Wolfe 1993; Djorgovski et al. 1996, and references therein).

Fontana et al. 1996), then, depending on the masses, these might be a serious problem for the open-bubble inflation model. On the other hand, the recent discovery of galaxy groups at $z \sim 2.4$ (e.g., Francis et al. 1996; Pascarelle et al. 1996) probably do not pose a serious threat for the open-bubble inflation model, while massive clusters at $z \sim 0.5 - 1$ (e.g., Luppino & Gioia 1995; Pelló et al. 1996) can easily be accommodated in the model. It should be noted that in adiabatic $\Omega_0 = 1$ models normalized to fit the present small-scale observations, e.g., fiducial CDM (with a normalization inconsistent with that from the DMR), or MDM, or tilted CDM (without a cosmological constant), it is quite difficult, if not impossible, to accommodate the above observational indications of early structure formation (e.g., Ma & Bertschinger 1994; Ostriker & Cen 1996).

With the recent improvements in observational capabilities, neoclassical cosmological tests hold great promise for constraining the world model. It might be significant that current constraints from these tests are consistent with that region of the open-bubble inflation model parameter space that is favoured by the large-scale structure constraints. These tests include the *HST* elliptical galaxy number counts test (Driver et al. 1996), an early application of the apparent magnitude-redshift test using Type Ia supernovae (Perlmutter et al. 1996), as well as analyses of the rate of gravitational lensing of quasars by foreground galaxies (e.g., Torres & Waga 1996; Kochanek 1996). It should be noted that these tests are also consistent with $\Omega_0 = 1$ models, and plausibly with a time-variable cosmological “constant” dominated spatially-flat model (e.g., Ratra & Quillen 1992; Torres & Waga 1996), but they do put pressure on the flat- Λ CDM model.

Smaller-scale CMB spatial anisotropy measurements will eventually significantly constrain the allowed range of model-parameter values. Fig. 24 compares the $1-\sigma$ range of CMB spatial anisotropy predictions for a few representative open-bubble inflation (as well as flat-space scale-invariant spectrum open) models to available CMB spatial anisotropy observational data. From a preliminary comparison of the predictions of DMR two-year data normalized open-bubble inflation models to available CMB anisotropy observational data, Ratra et al. (1995) concluded that the range of parameter space for the open-bubble inflation model that was favoured by the other observational data was also consistent with the small-scale CMB anisotropy data. This result was quantified by GRS, who also considered open-bubble inflation models normalized to the $\pm 1-\sigma$ values of the DMR two-year data (and hence considered open-bubble inflation models normalized at close to the DMR four-year data value, see Figs. 5 and 6 of GRS). GRS discovered that (given the uncertainties associated with the smaller-scale measurements) the $1-\sigma$ uncertainty in the value of the DMR normalization precludes determination of robust constraints on model-parameter values, although the range of model-parameter space for the open-bubble inflation model favoured by the analysis here was found to be consistent

with the smaller-scale CMB anisotropy observations, and $\Omega_0 \sim 0.1$ open-bubble inflation models were not favoured by the smaller-scale CMB anisotropy observational data (GRS, Figs. 5 and 6).²⁹ A detailed analysis of the UCSB South Pole 1994 CMB anisotropy data (Gundersen et al. 1995) by Ganga et al. (1996a) reaches a similar conclusion: at 1σ (assuming a gaussian marginal probability distribution) the data favours open-bubble inflation models with $\Omega_0 < 0.5$, while at 2σ the UCSB South Pole 1994 data is consistent with the predictions of the open-bubble, flat- Λ , and fiducial CDM inflation models.

6. DISCUSSION AND CONCLUSION

We have compared the DMR 53 and 90 GHz sky maps to a variety of open model CMB anisotropy angular spectra in order to infer the normalization of these open cosmogonical models. Our analysis explicitly quantifies the small shifts in the inferred normalization amplitudes due to: (1) the small differences between the galactic- and ecliptic-coordinate sky maps; (2) the inclusion or exclusion of the $\ell = 2$ moment in the analysis; and, (3) the faint high-latitude Galactic emission treatment. We have defined a maximal 2σ uncertainty range based on the extremal solutions of the normalization fits, and a maximal 1σ uncertainty range may be defined in a similar manner. For this maximal 1σ $Q_{\text{rms-PS}}$ range the fractional 1σ uncertainty, at fixed Ω_B and h (but depending on the assumed CMB anisotropy angular spectrum and model-parameter values), ranges between $\sim 10\%$ and $\sim 12\%$ ³⁰. (Compare this to the $\sim 8\%$, 1σ , uncertainty of eq. [19].) Since part of this

²⁹ The recent analysis of Hancock et al. (1996b) is generally consistent with these results. They conclude that $\Omega_0 \sim 0.7$ is favoured, but even at 1σ $0.3 \leq \Omega_0 \leq 1.7$ is allowed — this broad range is consistent with the conclusion of GRS that it is not yet possible to meaningfully constrain cosmological-parameter values from the CMB anisotropy data alone. Note also that Hancock et al. (1996b) do not consider the effects of the systematic shifts between the various DMR data sets, and also exclude a number of data points, e.g., the four MSAM points and the MAX3 MUP point (which is consistent with the recent MAX5 MUP result, Lim et al. 1996), which do not disfavour a lower value of Ω_0 for the open-bubble inflation model (Ratra et al. 1995; GRS).

³⁰ Note that the quoted 1σ (statistical and systematic) uncertainty of BW (footnote 4, also see Bunn, Liddle, & White 1996), 7.6%, is smaller than the DMR four-year data 1σ uncertainty estimated in, e.g., G96, Wright et al. (1996), and here. This is because we explicitly estimate the effect of all known systematic uncertainties for each assumed CMB anisotropy angular spectrum, and account for them, in the most conservative manner possible, as small shifts. (In particular: we do not just account for the small systematic difference between the galactic- and ecliptic-frame maps; we do not assume that any of the small systematic differences lead to model-independent systematic shifts in the inferred $Q_{\text{rms-PS}}$ values; and we do not add the systematic shifts in quadrature with the statistical uncertainty.) Since our accounting of the uncertainties is the most conservative possible, our conclusions about model-viability are the most robust possible.

uncertainty is due to the small systematic shifts, the maximal 2σ fractional uncertainty is smaller than twice the maximal 1σ fractional uncertainty. For the largest possible 2σ $Q_{\text{rms-PS}}$ range defined above, the fractional uncertainty varies between $\sim 16\%$ and $\sim 19\%$. Note that this accounts for intrinsic noise, cosmic variance, and effects (1)–(3) above. Other systematic effects, e.g., the calibration uncertainty (Kogut et al. 1996b), or the beamwidth uncertainty (Wright et al. 1994), are much smaller than the effects we have accounted for here. It has also been shown that there is negligible non-CMB contribution to the DMR data sets from known extragalactic astrophysical foregrounds (Banday et al. 1996b).

By analyzing the DMR maps using CMB anisotropy spectra at fixed Ω_0 but different h and Ω_B , we have also explicitly quantified the small shifts in the inferred normalization amplitude due to shifts in h and Ω_B . Although these shifts do depend on the value of Ω_0 and the assumed model power spectrum, given the other uncertainties, it is reasonable to ignore these small shifts when normalizing the models considered in this work.

We have analyzed the open-bubble inflation model, accounting only for the fluctuations generated during the evolution inside the bubble (RP94), including the effects of the fluctuations generated in the first epoch of spatially-flat inflation (BGT; YST), and finally accounting for the contribution from a non-square-integrable basis function (YST). For observationally viable open-bubble models, the observable predictions do not depend significantly on the latter two sources of anisotropy. The observable predictions of the open-bubble inflation scenario seem to be robust — it seems that only those fluctuations generated during the evolution inside the bubble need to be accounted for.

As discussed in the Introduction, a variety of more specific realizations of the open-bubble inflation scenario have recently come under scrutiny. These are based on specific assumptions about the vacuum state prior to open-bubble nucleation. In these specific realizations of the open-bubble inflation scenario there are a number of additional mechanisms for stress-energy perturbation generation (in addition to those in the models considered here), including those that come from fluctuations in the bubble wall, as well as effects associated with the nucleation of a nonzero size bubble. While current analyses suggest that such effects also do not add a significant amount to the fluctuations generated during the evolution inside the bubble, it is important to continue to pursue such investigations — both to more carefully examine the robustness of the open-bubble inflation scenario predictions, as well as to try to find a reasonable particle physics based realization of the open-bubble inflation scenario.

As has been previously noted for other CMB anisotropy angular spectra (G96), the various different DMR data sets lead to slightly different $Q_{\text{rms-PS}}$ normalization amplitudes, but well within the statistical uncertainty. This total range is slightly reduced if one

considers results from analyses either ignoring or including the quadrupole moment.

The DMR data alone can not be used to constrain Ω_0 over range $0.1 \leq \Omega_0 \leq 1$ in a statistically meaningful fashion for the open models considered here. It is, however, reasonable to conclude that when the quadrupole moment is excluded from the analysis, the $\Omega_0 \sim 0.4$ model CMB anisotropy spectral shape is most consistent with the DMR data, while the quadrupole-included analysis favours $\Omega_0 \sim 0.7$ (for the open-bubble inflation model in the range $0.1 \leq \Omega_0 \leq 1$).

Current cosmographic observations, in conjunction with current large-scale structure observations compared to the predictions of the DMR-normalized open-bubble inflation model derived here, favour $0.3 < \Omega_0 \lesssim 0.6$. The large allowed range is partially a consequence of the current uncertainty in Ω_B . This range is consistent with the value weakly favoured ($\Omega_0 \sim 0.4$) by a quadrupole-excluded analysis of the DMR data alone. It might also be significant that mild bias is indicated both by the need to reconcile these larger values of Ω_0 with what is determined from small-scale dynamical estimates, as well as to reconcile the smaller DMR-normalized $(\delta M/M)[8h^{-1} \text{ Mpc}]$ values (for this favoured range of Ω_0) with the larger observed galaxy number fluctuations (e.g., eq. [18]).

In common with the low-density flat- Λ CDM model, we have established that in the low-density open-bubble CDM model one may adjust the value of Ω_0 to accommodate a large fraction of present observational constraints. For a broad class of these models, with adiabatic gaussian initial energy-density perturbations, this focuses attention on values of Ω_0 that are larger than the range of values for Ω_B inferred from the observed light-element abundances in conjunction with standard nucleosynthesis theory. Whether this additional CDM is nonbaryonic, or is simply baryonic material that does not take part in standard nucleosynthesis, remains a major outstanding puzzle for these models.

In conclusion, the open-bubble inflation model with $0.3 < \Omega_0 \lesssim 0.6$ is most consistent with current observations.

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FIGURE CAPTIONS

Fig. 1.–Fractional differences, $\Delta C_\ell/C_\ell$, between the CMB spatial anisotropy multipole coefficients C_ℓ computed using the two Boltzmann transfer codes (and normalized to agree at $\ell = 9$). Heavy type is for the open-bubble inflation model spectrum accounting only for perturbations that are generated during the evolution inside the bubble (type [1] spectra above), and light type is for the open-bubble inflation model spectrum now also accounting for perturbations generated in the first epoch of inflation (type [2] spectra). Solid lines are for $\Omega_0 = 0.2$ and dashed lines are for $\Omega_0 = 0.5$. These are for $h = 0.6$ and $\Omega_B = 0.035$. Note that $\Delta Q_{\text{rms-PS}}/Q_{\text{rms-PS}} \simeq 0.5\Delta C_2/C_2$.

Fig. 2.–(a) CMB anisotropy multipole coefficients for the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94, solid lines), and also accounting for fluctuations generated in the first epoch of inflation (BGT; YST, dotted lines — these overlap the solid lines, except at the lowest Ω_0 and smallest ℓ), for $\Omega_0 = 0.1, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.6, 0.8$, and 1.0 , in ascending order. These are for $t_0 \simeq 12$ Gyr and $\Omega_B h^2 = 0.0125$. The coefficients are normalized relative to the C_9 amplitude, and different values of Ω_0 are offset from each other to aid visualization. In (b) are the set of CMB anisotropy spectra for the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94), with $\Omega_0 = 0.2$ and $\Omega_0 = 0.5$ for the three different pairs of values ($t_0, \Omega_B h^2$): ($\simeq 10.5$ Gyr, $= 0.0055$), ($\simeq 12$ Gyr, $= 0.0125$), and ($\simeq 13.5$ Gyr, $= 0.0205$). Spectra in the two sets are normalized to have the same C_9 , and $\Omega_B h^2$ increases in ascending order on the right axis.

Fig. 3.–CMB spatial anisotropy multipole coefficients for the flat-space scale-invariant spectrum open model (W83). Conventions and parameter values are as in the caption of Fig. 2 (although only one set of spectra are shown in Fig. 3a).

Fig. 4.–CMB spatial anisotropy multipole coefficients for the open-bubble inflation spectrum, also accounting for both fluctuations generated in the first epoch of inflation and that corresponding to a non-square-integrable basis function (YST, solid lines), and ignoring both these fluctuations (RP94, dotted lines). They are, in ascending order, for $\Omega_0 = 0.1$ to 0.9 in steps of 0.1 , with $h = 0.6$ and $\Omega_B = 0.035$, normalized relative to the C_9 amplitude, and different values of Ω_0 are offset from each other to aid visualization.

Fig. 5.–CMB spatial anisotropy multipole coefficients, as a function of ℓ , for the various

spectra considered in this paper, at $\Omega_0 = 0.2$ and 0.5 (vertically offset). Light solid and heavy solid lines show the open-bubble inflation cases accounting for (type [2] spectra above) and ignoring (type [1] spectra, at $\Omega_0 = 0.5$ these completely overlap the type [2] spectra) fluctuations generated in the first epoch of inflation. Dashed lines show the open-bubble inflation models, now also accounting for the contribution from the non-square-integrable basis function (type [3] spectra). Dotted lines show the flat-space scale-invariant spectrum open model spectra (type [4] spectra). All spectra are for $h = 0.6$ and $\Omega_B = 0.035$.

Fig. 6.–Likelihood functions $L(Q_{\text{rms-PS}}, \Omega_0)$ (arbitrarily normalized to unity at the highest peak at $\Omega_0 \sim 0.4$) derived from a simultaneous analysis of the DMR 53 and 90 GHz ecliptic-frame data, ignoring the correction for faint high-latitude foreground Galactic emission, and excluding the quadrupole moment from the analysis. These are for the $h = 0.6$, $\Omega_B = 0.035$ models. Panel (a) is for the flat-space scale-invariant spectrum open model (W83), (b) is for the open-bubble inflation model accounting only for perturbations generated during the evolution inside the bubble (RP94), and (c) is for the open-bubble inflation model now also accounting for both the fluctuations generated in the first epoch of inflation and those corresponding to a non-square-integrable basis function (YST).

Fig. 7.–Likelihood functions $L(Q_{\text{rms-PS}}, \Omega_0)$ (arbitrarily normalized to unity at the highest peak near either $\Omega_0 \sim 0.1$ or 0.7), derived from a simultaneous analysis of the DMR 53 and 90 GHz galactic-frame data, accounting for the faint high-latitude foreground Galactic emission correction, and including the quadrupole moment in the analysis. Conventions and parameter values are as for Fig. 6.

Fig. 8.–Ridge lines of the maximum likelihood $Q_{\text{rms-PS}}$ value as a function of Ω_0 , for the open-bubble inflation model accounting only for fluctuations generated during the evolution inside the bubble (type [1] spectra), for the eight different DMR data sets considered here, and for $t_0 \simeq 12$ Gyr, $\Omega_B h^2 = 0.0125$. Heavy lines correspond to the case when the quadrupole moment is excluded from the analysis, while light lines account for the quadrupole moment. These are for the ecliptic-frame sky maps, accounting for (dashed lines) and ignoring (solid lines) the faint high-latitude foreground Galactic emission correction, and for the galactic-frame maps, accounting for (dot-dashed lines) and ignoring (dotted lines) this Galactic emission correction. The general features of this figure are consistent with that derived from the DMR two-year data (GRSB, Fig. 2).

Fig. 9.–Ridge lines of the maximum likelihood $Q_{\text{rms-PS}}$ value as a function of Ω_0 , for the flat-space scale-invariant spectrum open model (type [4] spectra), for the eight different DMR data sets, and for $t_0 \simeq 12$ Gyr, $\Omega_B h^2 = 0.0125$. Heavy lines correspond to the

ecliptic-frame analyses, while light lines are from the galactic-frame analyses. These are for the cases ignoring the faint high-latitude foreground Galactic-emission correction, and either including (dotted lines) or excluding (solid lines) the quadrupole moment; and accounting for this Galactic emission correction, and either including (dot-dashed lines) or excluding (dashed lines) the quadrupole moment. The general features of this figure are roughly consistent with that derived from the DMR two-year data (Cayón et al. 1996, Fig. 3).

Fig. 10.–Ridge lines of the maximum likelihood $Q_{\text{rms-PS}}$ value as a function of Ω_0 , for the open-bubble inflation model now also accounting for both the fluctuations generated in the first epoch of inflation (BGT; YST) and those from a non-square-integrable basis function (YST), for the eight different DMR data sets considered here, and for $h = 0.6$, $\Omega_B = 0.035$. Heavy lines correspond to the cases where the faint high-latitude foreground Galactic emission correction is ignored, while light lines account for this Galactic emission correction. These are from the ecliptic frame analyses, accounting for (dotted lines) or ignoring (solid lines) the quadrupole moment; and from the galactic-frame analyses, accounting for (dot-dashed lines) or ignoring (dashed lines) the quadrupole moment. The general features of this figure are consistent with that derived from the DMR two-year data (YB, Fig. 2).

Fig. 11.–Ridge lines of the maximum likelihood $Q_{\text{rms-PS}}$ value as a function of Ω_0 , for the two extreme DMR data sets, and two different CMB anisotropy angular spectra, showing the effects of varying t_0 and $\Omega_B h^2$. Heavy lines are for $t_0 \simeq 13.5$ Gyr and $\Omega_B h^2 = 0.0205$, while light lines are for $t_0 \simeq 10.5$ Gyr and $\Omega_B h^2 = 0.0055$. Two of the four pairs of lines are for the open-bubble inflation model accounting only for fluctuations generated during the evolution inside the bubble (type [1] spectra), either from the ecliptic-frame analysis without the faint high-latitude foreground Galactic emission correction and ignoring the quadrupole moment in the analysis (solid lines), or from the galactic-frame analysis accounting for this Galactic emission correction and including the quadrupole moment in the analysis (dotted lines). The other two of the four pairs of lines are for the flat-space scale-invariant spectrum open model (type [4] spectra), either from the ecliptic-frame analysis without the faint high-latitude foreground Galactic emission correction and ignoring the quadrupole moment in the analysis (dashed lines), or from the galactic-frame analysis accounting for this Galactic emission correction and including the quadrupole moment in the analysis (dot-dashed lines). Given the other uncertainties, the effects of varying t_0 and $\Omega_B h^2$ are fairly negligible.

Fig. 12.–Ridge lines of the maximum likelihood $Q_{\text{rms-PS}}$ value as a function of Ω_0 , for the two extreme DMR data sets, for the four CMB anisotropy angular spectra models

considered here, and for $h = 0.6$, $\Omega_B = 0.035$. Heavy lines are from the ecliptic-frame sky maps ignoring the faint high-latitude foreground Galactic emission correction and excluding the quadrupole moment from the analysis, while light lines are from the galactic-frame sky maps accounting for this Galactic emission correction and including the quadrupole moment in the analysis. Solid, dotted, and dashed lines show the open-bubble inflation cases, accounting only for the fluctuations generated during the evolution inside the bubble (type [1] spectra, solid lines), also accounting for the fluctuations generated in the first epoch of inflation (type [2] spectra, dotted lines — these overlap the solid lines except for $\Omega_0 \lesssim 0.2$ and $\Omega_0 \sim 0.7$), and finally also accounting for the fluctuations corresponding to the non-square-integrable basis function (type [3] spectra, dashed lines). Dot-dashed lines correspond to the flat-space scale-invariant spectrum open model (type [4] spectra).

Fig. 13.—Conditional likelihood densities for $Q_{\text{rms-PS}}$, derived from $L(Q_{\text{rms-PS}}, \Omega_0)$ (which are normalized to be unity at the peak, for each DMR data set, CMB anisotropy angular spectrum, and set of model-parameter values). Panel (a) is for the open-bubble inflation model accounting only for fluctuations generated during the evolution inside the bubble (type [1] spectra), while panel (b) is for the flat-space scale-invariant spectrum open model (type [4] spectra). The heavy lines are for $\Omega_0 = 0.2$, while the light lines are for $\Omega_0 = 0.5$. Two of the four pairs of lines in each panel correspond to the results from the analysis of the galactic-frame maps accounting for the faint high-latitude foreground Galactic emission correction and with the quadrupole moment included in the analysis, either for $t_0 \simeq 10.5$ Gyr and $\Omega_B h^2 = 0.0055$ (dot-dashed lines), or for $t_0 \simeq 13.5$ Gyr and $\Omega_B h^2 = 0.0205$ (dashed lines). The other two pairs of lines in each panel correspond to the results from the analysis of the ecliptic-frame maps ignoring this Galactic emission correction and with the quadrupole moment excluded from the analysis, either for $t_0 \simeq 10.5$ Gyr and $\Omega_B h^2 = 0.0055$ (dotted lines), or for $t_0 \simeq 13.5$ Gyr and $\Omega_B h^2 = 0.0205$ (solid lines). Given the other uncertainties, the effects of varying t_0 and $\Omega_B h^2$ are fairly negligible.

Fig. 14.—Conditional likelihood densities for $Q_{\text{rms-PS}}$ normalized as in the caption for Fig. 13. Panel (a) is from the analysis of the ecliptic-frame maps ignoring the faint high-latitude foreground Galactic emission correction and excluding the quadrupole moment from the analysis, while panel (b) is from the analysis of the galactic-frame maps accounting for this Galactic emission correction and including the quadrupole moment in the analysis. These are for $h = 0.6$ and $\Omega_B = 0.035$. The heavy lines are for $\Omega_0 = 0.2$ and the light lines are for $\Omega_0 = 0.5$. There are eight lines (four pairs) in each panel, although in each panel two pairs almost identically overlap. Solid, dotted, and dashed lines show the open-bubble inflation cases, accounting only for the fluctuations generated during the evolution inside the bubble (type [1] spectra, solid lines), also accounting for the fluctuations generated

in the first epoch of inflation (type [2] spectra, dotted lines — these almost identically overlap the solid lines), and finally also accounting for the fluctuations corresponding to the non-square-integrable basis function (type [3] spectra, dashed lines). Dot-dashed lines correspond to the flat-space scale-invariant spectrum open model (type [4] spectra).

Fig. 15.—Projected likelihood densities for Ω_0 derived from $L(Q_{\text{rms-PS}}, \Omega_0)$ (normalized as in the caption of Fig. 13). Panel (a) is for the open-bubble inflation model accounting only for the fluctuations generated during the evolution inside the bubble (type [1] spectra), and panel (b) is for the flat-space scale-invariant spectrum open model (type [4] spectra). Two of the curves in each panel correspond to the results from the analysis of the galactic-frame maps accounting for the faint high-latitude foreground Galactic emission correction and with the quadrupole moment included in the analysis, for $t_0 \simeq 10.5$ Gyr and $\Omega_B h^2 = 0.0055$ (dot-dashed lines) and for $t_0 \simeq 13.5$ Gyr and $\Omega_B h^2 = 0.0205$ (dashed lines). The other two curves in each panel are from the analysis of the ecliptic-frame maps ignoring the Galactic emission correction and excluding the quadrupole moment from the analysis, for $t_0 \simeq 10.5$ Gyr and $\Omega_B h^2 = 0.0055$ (dotted lines) and for $t_0 \simeq 13.5$ Gyr and $\Omega_B h^2 = 0.0205$ (solid lines).

Fig. 16.—Projected likelihood densities for Ω_0 derived from $L(Q_{\text{rms-PS}}, \Omega_0)$ (normalized as in the caption of Fig. 13). Panel (a) is from the analysis of the ecliptic-frame sky maps ignoring the faint high-latitude foreground Galactic emission correction and excluding the quadrupole moment from the analysis. Panel (b) is from the analysis of the galactic-frame sky maps accounting for this Galactic emission correction and including the quadrupole moment in the analysis. There are four curves in each panel, although in each panel two of them almost overlap. Solid, dotted, and dashed lines show the open-bubble inflation cases, accounting only for the fluctuations generated during the evolution inside the bubble (type [1] spectra, solid lines), also accounting for the fluctuations generated in the first epoch of spatially-flat inflation (type [2] spectra, dotted lines — these almost exactly overlap the solid lines), and finally also accounting for the fluctuations corresponding to the non-square-integrable basis function (type [3] spectra, dashed lines). Dot-dashed lines correspond to the flat-space scale-invariant spectrum open model (type [4] spectra). These are for $h = 0.6$ and $\Omega_B = 0.035$.

Fig. 17.—Marginal likelihood densities [$\propto \int dQ_{\text{rms-PS}} L(Q_{\text{rms-PS}}, \Omega_0)$] for Ω_0 , normalized to unity at the peak, for the open-bubble inflation model accounting only for fluctuations generated during the evolution inside the bubble (RP94), for the eight different DMR data sets, and for $t_0 \simeq 12$ Gyr, $\Omega_B h^2 = 0.0125$. Panel (a) is from the ecliptic-frame analyses, and panel (b) is from the galactic-frame analyses. Two of the four lines in each panel are from

the analysis without the faint high-latitude foreground Galactic emission correction, either accounting for (dot-dashed lines) or ignoring (solid lines) the quadrupole moment. The other two lines in each panel are from the analysis with this Galactic emission correction, either accounting for (dotted lines) or ignoring (dashed lines) the quadrupole moment.

Fig. 18.–Marginal likelihood densities for Ω_0 , for the flat-space scale-invariant spectrum open model (W83). Conventions and parameter values are as in the caption of Fig. 17.

Fig. 19.–Marginal likelihood densities for Ω_0 , for the open-bubble inflation model now also accounting for both the fluctuations generated in the first spatially-flat epoch of inflation and those that correspond to the non-square-integrable basis function (YST), computed for $h = 0.6$ and $\Omega_B = 0.035$. Conventions are as in the caption of Fig. 17.

Fig. 20.–Marginal likelihood densities for Ω_0 (normalized as in the caption of Fig. 17). Panel (a) is for the open-bubble inflation model accounting only for the fluctuations generated during the evolution inside the bubble (RP94), while panel (b) is for the flat-space scale-invariant spectrum open model (W83). Two of the lines in each panel are the results from the analysis of the galactic-frame data sets accounting for the faint high-latitude foreground Galactic emission correction and with the quadrupole moment included in the analysis, for $t_0 \simeq 10.5$ Gyr and $\Omega_B h^2 = 0.0055$ (dot-dashed lines), and for $t_0 \simeq 13.5$ Gyr and $\Omega_B h^2 = 0.0205$ (dashed lines). The other two lines in each panel are the results from the analysis of the ecliptic-frame data sets ignoring this Galactic emission correction and with the quadrupole moment excluded from the analysis, for $t_0 \simeq 10.5$ Gyr and $\Omega_B h^2 = 0.0055$ (dotted lines), and for $t_0 \simeq 13.5$ Gyr and $\Omega_B h^2 = 0.0205$ (solid lines).

Fig. 21.–Marginal likelihood densities for Ω_0 (normalized as in the caption of Fig. 17), computed for $h = 0.6$ and $\Omega_B = 0.035$. Panel (a) is from the analysis of the ecliptic-frame sky maps ignoring the faint high-latitude foreground Galactic emission correction and excluding the quadrupole moment from the analysis. Panel (b) is from the analysis of the galactic-frame sky maps accounting for this Galactic emission correction and including the quadrupole moment in the analysis. There are four lines in each panel, although in each panel two of the lines almost overlap. Solid, dotted, and dashed curves are the open-bubble inflation cases, accounting only for the fluctuations generated during the evolution inside the bubble (RP94, solid lines), also accounting for the fluctuations generated in the first epoch of spatially-flat inflation (BGT; YST, dotted lines — these almost identically overlap the solid lines), and finally also accounting for the fluctuations corresponding to the non-square-integrable basis function (YST, dashed lines). Dot-dashed curves correspond to the flat-space scale-invariant spectrum open model (W83).

Fig. 22.–Fractional differences, $\Delta P(k)/P(k)$, as a function of wavenumber k , between the energy-density perturbation power spectra $P(k)$ computed using the two independent numerical integration codes (and normalized to give the same $Q_{\text{rms-PS}}$). The heavy curves are for the open-bubble inflation model spectrum accounting only for fluctuations that are generated during the evolution inside the bubble (type [1] spectra), and the light curves are for the open-bubble inflation model spectrum now also accounting for fluctuations generated in the first epoch of inflation (type [2] spectra). These are for $\Omega_0 = 0.2$ (solid lines) and $\Omega_0 = 0.5$ (dashed lines), with $h = 0.6$ and $\Omega_B = 0.035$.

Fig. 23.–Fractional energy-density perturbation power spectra $P(k)$ as a function of wavenumber k . These are normalized to the mean of the extreme upper and lower 2σ $Q_{\text{rms-PS}}$ values (as discussed in §3.3). Panels (a)–(d) correspond to the four different sets of $(t_0, \Omega_B h^2)$ of Tables 9–12, and each panel shows power spectra for three different models at six values of Ω_0 . Solid lines show the open-bubble inflation model $P(k)$ accounting only for fluctuations generated during the evolution inside the bubble (RP95); dotted lines are for the open-bubble inflation model now also accounting for fluctuations generated in the first epoch of inflation (BGT; YST); and, dashed lines are for the flat-space scale-invariant spectrum open model (W83). Starting near the center of the lower horizontal axis, and moving counterclockwise, the spectra shown correspond to $\Omega_0 = 0.1, 0.2, 0.3, 0.45, 0.6$, and 1. Note that at $\Omega_0 = 1$ all three model spectra are identical and so overlap; also note that at a given Ω_0 the open-bubble inflation model $P(k)$ accounting for the fluctuations generated in the first epoch of inflation (BGT; YST, dotted lines) essentially overlap those where this source of fluctuations is ignored (RP95, solid lines). Panel (a) corresponds to $t_0 \simeq 10.5$ Gyr and $\Omega_B h^2 = 0.0055$, (b) to $t_0 \simeq 12$ Gyr and $\Omega_B h^2 = 0.0125$, (c) to $t_0 \simeq 13.5$ Gyr and $\Omega_B h^2 = 0.205$, and (d) to $t_0 \simeq 12$ Gyr and $\Omega_B h^2 = 0.007$ (normalized using the results of the DMR analysis of the $t_0 \simeq 10.5$ Gyr, $\Omega_B h^2 = 0.0055$ models). Panel (e) shows the three $h = 0.6$, $\Omega_B = 0.035$ open-bubble inflation spectra of Table 13 at five different values of Ω_0 . The spectra are for the open-bubble inflation model accounting only for fluctuations generated during the evolution inside the bubble (RP95, solid lines), also accounting for fluctuations generated in the first epoch of inflation (BGT; YST, dotted lines), and also accounting for the contribution from the non-square-integrable basis function (YST, dashed lines). Starting near the center of the lower horizontal axis and moving counterclockwise, the models correspond to $\Omega_0 = 0.1, 0.2, 0.3, 0.5$, and 0.9. Note that at a given Ω_0 the three spectra essentially overlap, especially for observationally-viable values of $\Omega_0 \gtrsim 0.3$. The solid triangles represent the redshift-space da Costa et al. (1994) SSRS2 + CfA2 ($130 h^{-1}$ Mpc depth) optical galaxies data (and were very kindly provided to us by C. Park). The solid squares represent the [$P(k) = 8000(h^{-1} \text{ Mpc})^3$ weighting] redshift-space results of the Tadros & Efstathiou (1995) analysis of the *IRAS* QDOT and 1.2 Jy infrared galaxy

data. The hollow pentagons represent the real-space results of the Baugh & Efstathiou (1993) analysis of the APM optical galaxy data (and were very kindly provided to us by C. Baugh). It should be noted that the plotted model mass (not galaxy) power spectra do not account for any bias of galaxies with respect to mass. They also do not account for nonlinear or redshift-space-distortion (when relevant) corrections nor for the survey window functions. It should also be noted that the observational data error bars are determined under the assumption of a specific cosmological model and a specific evolution scenario, i.e., they do not necessarily account for these additional sources of uncertainty (e.g., Gaztañaga 1995). We emphasize that, because of the different assumptions, the different observed galaxy power spectra shown on the plots are defined somewhat differently and so cannot be directly quantitatively compared to each other.

Fig. 24.–CMB anisotropy bandtemperature predictions and observational results, as a function of multipole ℓ , to $\ell = 1000$. The four pairs of wavy curves (in different linestyles) demarcating the boundaries of the four partially overlapping wavy hatched regions (hatched with straight lines in different linestyles) in panel (a) are DMR-normalized open-bubble inflation model (RP94) predictions for what would be seen by a series of ideal, Kronecker-delta window-function, experiments (see Ratra et al. 1995 for details). Panel (b) shows DMR-normalized CMB anisotropy spectra with the same cosmological parameters for the flat-space scale-invariant spectrum open model (W83). The model-parameter values are: $\Omega_0 = 0.3$, $h = 0.7$, $\Omega_B h^2 = 0.0075$, $t_0 = 11.3$ Gyr (dot-dashed lines); $\Omega_0 = 0.4$, $h = 0.65$, $\Omega_B h^2 = 0.0125$, $t_0 = 11.7$ Gyr (solid lines); $\Omega_0 = 0.5$, $h = 0.55$, $\Omega_B h^2 = 0.0175$, $t_0 = 13.4$ Gyr (dashed lines); and, $\Omega_0 = 1$, $h = 0.5$, $\Omega_B h^2 = 0.0125$, $t_0 = 13.0$ Gyr (dotted lines) — for more details on these models see Ratra et al. (1995). For each pair of model-prediction demarcation curves, the lower one is normalized to the lower $1-\sigma$ $Q_{\text{rms-PS}}$ value determined from the analysis of the galactic-coordinate maps accounting for the high-latitude Galactic emission correction and including the $\ell = 2$ moment in the analysis, and the upper one is normalized to the upper $1-\sigma$ $Q_{\text{rms-PS}}$ value determined from the analysis of the ecliptic-coordinate maps ignoring the Galactic emission correction and excluding the $\ell = 2$ moment from the analysis. Amongst the open-bubble inflation models of panel (a), the $\Omega_0 = 0.4$ model is close to what is favoured by the analysis of Table 10, and the $\Omega_0 = 0.5$ model is close to that preferred from the analysis of Table 11. The $\Omega_0 = 0.3$ model is on the edge of the allowed region from the analysis of Table 12, and the $\Omega_0 = 1$ fiducial CDM model is incompatible with cosmographic and large-scale structure observations. A large fraction of the smaller-scale observational data in these plots are tabulated in Ratra et al. (1995) and Ratra & Sugiyama (1995). Note that, as discussed in these papers, some of the data points are from reanalyses of the observational data. There are 69 detections and 22 $2-\sigma$ upper limits shown. Since most of the smaller-scale data points are derived

assuming a flat bandpower CMB anisotropy angular spectrum, which is more accurate for narrower (in ℓ) window functions, we have shown the observational results from the narrowest windows available. The data shown are from the DMR galactic frame maps ignoring the Galactic emission correction (Górski 1996, open octagons with $\ell \leq 20$); from FIRS (Ganga et al. 1994, as analyzed by Bond 1995, solid pentagon); Tenerife (Hancock et al. 1996a, open five-point star); Bartol (Piccirillo et al. 1996, solid diamond, note that atmospheric contamination may be an issue); SK93, individual-chop SK94 Ka and Q, and individual-chop SK95 cap and ring (Netterfield et al. 1996, open squares); SP94 Ka and Q (Gundersen et al. 1995, the points plotted here are from the flat bandpower analysis of Ganga et al. 1996a, solid circles); BAM 2-beam (Tucker et al. 1996, at $\ell_{\text{eff}} = 58.2$ with $\ell_{e-0.5}$ spanning 16 to 92, and accounting for the 20% calibration uncertainty, open circle); Python-G, -L, and -S (e.g., Platt et al. 1996, open six-point stars); ARGO (e.g., Masi et al. 1996, both the Hercules and Aries+Taurus scans are shown — note that the Aries+Taurus scan has a larger calibration uncertainty of 10%, solid squares); MAX3, individual-channel MAX4, and MAX5 (e.g., Tanaka et al. 1996, including the MAX5 MUP 2- σ upper limit $\delta T_\ell < 35 \mu\text{K}$ at $\ell_{\text{eff}} = 139$, Lim et al. 1996, open hexagons); MSAM92 and MSAM94 (e.g., Inman et al. 1996, open diamonds); WDH1–3 and WDI, II (e.g., Griffin et al. 1996, open pentagons); and CAT (Scott et al. 1996 — CAT1 at $\ell_{\text{eff}} = 396$ with $\ell_{e-0.5}$ spanning 351 to 471, and CAT2 at $\ell_{\text{eff}} = 608$ with $\ell_{e-0.5}$ spanning 565 to 710, both accounting for calibration uncertainty of 5%, solid hexagons). Detections have vertical 1- σ error bars. Solid inverted triangles inserted inside the appropriate symbols correspond to nondetections, and are placed at the upper 2- σ limits. Vertical error bars are not shown for non-detections. As discussed in Ratra et al. (1995), all δT_ℓ (vertical) error bars also account for the calibration uncertainty (but in an approximate manner, except for the SP94 Ka and Q results from Ganga et al. 1996a — see Ganga et al. 1996a for a discussion of this issue). The observational data points are placed at the ℓ -value at which the corresponding window function is most sensitive (this ignores the fact that the sensitivity of the experiment is also dependent on the assumed form of the sky-anisotropy signal, and so gives a somewhat misleading impression of the multipoles to which the experiment is sensitive — see Ganga et al. 1996a for a discussion of this issue). Excluding the DMR points at $\ell \leq 20$, the horizontal lines on the observational data points represent the ℓ -space width of the corresponding window function (again ignoring the form of the sky-anisotropy signal). Note that from an analysis of a large fraction of the data (corresponding to detections of CMB anisotropy) shown in these figures, GRS (Figs. 5 and 6) conclude that all the models shown in panel (a), including the fiducial CDM one, are consistent with the CMB anisotropy data.

TABLE 1
 $Q_{\text{rms-PS}}$ Values for the $t_0 \simeq 10.5$ Gyr, $\Omega_B h^2 = 0.0055$ Open-Bubble Inflation Models^a

Frame:		Ecliptic					Galactic																			
G.C. ^b :		No	No	Yes	Yes	No	No	Yes	Yes																	
Ω_0	h	$Q_{\text{rms-PS}}$ (μK)																								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)																	
0.1	0.84	21.04 19.51	22.61 18.12	21.56 20.03	23.18 18.58	24.93 18.86	20.40 17.51	21.99 17.86	23.61 17.86	22.42 19.25	24.12 17.86	20.39 18.91	21.92 17.52	23.63 17.94	20.90 19.37	22.47 17.94	24.19 16.96	19.77 18.33	21.31 16.96	22.97 20.20	20.20 18.68	21.73 17.29	23.44 20.20			
0.2	0.79	24.28 22.52	26.04 20.90	24.89 23.10	26.72 21.44	28.72 21.81	23.58 20.18	25.25 22.33	27.20 20.70	24.09 22.33	25.89 20.70	23.49 21.78	25.25 20.20	27.19 20.71	24.09 22.33	25.90 20.71	27.84 20.71	22.80 21.09	24.50 19.52	26.42 20.02	23.31 21.59	25.11 20.02	27.01 20.02			
0.25	0.77	24.28 22.56	26.04 20.94	24.84 23.10	26.67 21.48	28.63 21.85	23.63 20.27	25.25 22.32	27.10 20.70	24.10 22.32	25.84 20.70	23.49 21.82	25.25 20.25	27.15 20.76	24.09 22.33	25.85 20.76	27.80 20.76	22.80 21.14	24.50 19.60	26.38 20.02	23.31 21.59	25.06 20.02	26.96 20.02			
0.3	0.75	23.63 22.01	25.34 20.48	24.20 22.50	27.19 20.97	27.82 19.80	22.96 21.39	24.69 19.80	26.40 20.24	23.44 21.83	25.21 20.24	22.89 21.27	24.56 19.79	26.41 19.79	23.44 21.78	25.16 20.25	27.01 20.25	22.24 20.63	23.86 19.14	25.65 20.56	22.70 21.08	24.42 19.56	26.22 20.02			
0.35	0.74	22.61 21.04	24.19 19.60	23.10 21.52	25.94 20.07	26.55 19.45	21.99 19.01	24.76 23.52	25.29 25.29	22.43 20.82	25.76 19.41	21.92 20.39	23.49 18.95	25.20 18.95	22.43 20.90	24.05 19.42	25.81 19.42	21.26 19.77	22.84 18.37	24.54 18.37	21.73 20.16	23.31 18.77	25.02 18.77			
0.4	0.73	21.36 19.88	22.84 18.54	21.82 20.37	24.46 18.96	25.01 18.96	20.77 19.28	22.21 17.90	23.73 17.90	21.18 19.68	22.66 18.27	20.71 19.28	22.19 17.98	23.81 17.98	21.18 19.74	22.70 18.35	24.32 18.35	20.11 18.71	21.56 17.43	23.14 20.09	20.53 17.75	22.01 17.75	23.63 20.09			
0.45	0.71	20.02 18.63	21.36 17.43	20.46 19.05	22.89 17.77	23.35 17.77	19.45 18.11	21.82 16.95	23.35 16.95	19.78 18.44	22.29 17.21	19.42 18.07	20.76 16.87	22.29 16.87	19.83 18.49	21.22 17.24	22.75 17.24	18.88 17.56	20.20 16.37	21.69 16.37	22.06 16.64	19.19 17.89	20.57 16.64	22.06 16.64		
0.5	0.70	18.68 17.43	19.93 16.27	19.05 17.81	21.31 16.62	21.78 15.83	18.21 16.87	19.28 16.87	20.74 15.83	18.51 17.15	19.65 16.05	18.12 16.92	19.42 15.76	20.76 15.76	21.11 15.76	18.54 17.24	19.79 16.13	21.18 16.13	17.60 16.45	18.88 15.30	20.24 15.30	17.94 16.69	19.19 15.53	20.57 15.53		
0.6	0.68	16.50 15.39	17.61 14.37	16.84 15.73	18.81 14.66	19.18 13.95	16.05 14.86	17.05 13.95	18.38 13.95	16.31 15.11	17.34 14.13	16.04 14.97	17.15 13.95	18.31 13.95	17.47 14.23	18.68 14.23	17.47 14.23	16.36 15.25	17.47 14.23	18.68 14.23	15.60 14.54	16.66 13.56	17.86 13.56	15.85 14.74	16.92 13.72	18.12 13.72
0.8	0.65	15.76 14.74	16.82 13.77	16.07 15.00	17.94 14.02	18.28 14.02	15.40 14.38	16.42 13.45	17.54 13.45	15.58 14.52	16.65 13.60	15.30 14.32	16.36 13.35	17.47 13.35	16.99 13.35	17.47 13.35	16.62 14.60	16.69 13.63	17.84 13.63	14.92 13.94	15.94 13.00	17.05 13.00	15.11 14.09	16.18 13.17	17.29 13.17	
1	0.62	18.07 16.87	19.28 15.76	18.41 17.22	20.57 16.11	21.01 16.11	17.68 16.48	18.89 15.26	20.00 15.26	17.91 16.70	19.11 15.51	18.72 15.51	17.52 16.36	18.06 15.30	20.06 15.30	17.89 16.73	19.14 15.62	20.48 15.62	17.09 16.73	18.28 15.62	19.52 14.83	17.33 15.06	18.54 15.06	19.83 15.06		

^aFor the open-bubble inflation model accounting only for fluctuations generated during the evolution inside the bubble (RP94). The tabulated $Q_{\text{rms-PS}}$ values are determined from the conditional likelihood function at fixed Ω_0 . At each Ω_0 , the first of the five entries in each of columns (3)–(10) is the maximum likelihood value, the first (vertical) pair delimits the 68.3% (1- σ) highest posterior density range, and the second (vertical) pair delimits the 95.5% (2- σ) highest posterior density range. ^bAccounting for (Yes), or ignoring (No) the correction for faint high-latitude foreground Galactic emission.

TABLE 2
 $Q_{\text{rms-PS}}$ Values for the $t_0 \simeq 12$ Gyr, $\Omega_B h^2 = 0.0125$ Open-Bubble Inflation Models^a

Frame:		Ecliptic					Galactic																		
G.C. ^b :		No	No	Yes	Yes	No	No	Yes	Yes																
ℓ_{min} :		2	3	2	3	2	3	2	3																
Ω_0	h	$Q_{\text{rms-PS}}$ (μK)																							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)																
0.1	0.73	21.04 19.56	22.66 18.12	21.56 20.03	23.18 18.58	24.93 17.57	20.40 18.96	22.08 17.94	23.66 19.37	24.18 17.94	20.39 18.91	21.92 17.52	23.63 17.98	20.90 19.37	22.47 17.98	24.23 18.33	21.31 16.96	22.97 20.20	21.78 18.72	23.49 17.33					
0.2	0.69	24.28 22.52	26.04 20.90	24.89 23.10	26.72 21.44	28.68 20.22	23.67 21.81	25.25 20.22	27.10 20.65	24.10 22.33	25.84 20.20	27.78 20.71	24.09 22.33	25.90 20.71	27.84 20.71	22.80 21.09	24.50 19.56	26.38 23.26	25.06 21.59	27.01 19.97					
0.25	0.67	24.23 22.52	25.99 20.94	24.84 23.10	26.63 21.48	28.59 21.48	23.58 21.85	25.20 20.15	27.16 20.68	24.04 22.33	25.79 20.68	27.79 20.68	23.44 21.78	25.20 20.25	27.10 20.71	24.05 22.33	25.81 20.71	27.75 20.71	22.80 21.14	24.46 19.56	26.33 23.26	25.02 21.59	26.96 20.02		
0.3	0.66	23.58 21.96	25.30 20.44	24.16 22.46	25.86 20.92	27.74 19.81	22.80 21.39	24.65 19.81	26.51 20.21	23.32 21.80	25.12 20.21	27.02 20.21	22.89 21.22	24.51 19.74	26.31 19.74	23.40 21.78	25.11 20.25	26.96 20.58	25.61 19.09	23.82 20.58	25.61 19.09	22.20 21.04	24.32 19.51	26.18 21.32	
0.35	0.65	22.56 20.99	24.14 19.56	23.05 21.48	24.67 20.03	26.46 19.35	21.94 18.84	25.46 18.84	25.09 19.29	22.34 20.80	23.95 19.29	25.65 19.29	21.87 20.34	23.44 18.95	25.16 18.95	22.38 20.81	23.95 19.37	25.71 19.37	22.80 19.73	24.46 18.33	24.46 21.64	23.26 20.16	24.97 18.72		
0.4	0.63	21.27 19.83	22.75 18.49	21.73 20.29	23.27 18.92	24.93 17.80	20.67 19.22	22.16 17.80	23.75 17.80	21.04 19.58	22.56 18.22	24.23 18.22	20.67 19.23	22.10 17.94	23.68 17.94	22.13 19.65	22.61 18.31	24.23 18.31	24.23 20.07	21.52 18.67	23.05 17.35	23.54 20.44	21.92 19.00	23.54 17.70	
0.45	0.62	19.93 18.58	21.27 17.33	20.37 18.96	21.73 18.96	23.27 17.73	19.36 18.01	20.60 16.80	22.20 16.80	19.73 18.34	21.01 17.14	22.61 17.14	19.32 18.34	20.71 16.78	22.15 16.78	19.74 18.40	21.13 17.15	22.61 17.15	21.56 17.47	22.61 16.28	21.11 16.28	21.56 17.47	21.96 16.59		
0.5	0.61	18.58 17.33	19.83 16.18	18.96 17.69	20.24 16.54	21.65 16.90	18.15 15.62	19.36 17.16	20.66 15.92	18.41 15.92	19.65 15.92	21.03 15.92	18.03 16.82	19.28 15.71	20.67 15.71	18.44 17.19	19.69 16.04	21.08 16.04	18.79 16.37	20.11 15.22	21.56 15.22	18.79 16.37	20.11 15.22	21.96 16.64	20.48 15.48
0.6	0.60	16.45 15.34	17.52 14.32	16.75 15.64	17.86 14.62	19.09 14.01	16.06 14.94	17.02 14.01	18.28 14.01	16.28 14.15	17.32 14.15	18.54 14.15	15.94 14.88	17.06 13.86	18.21 13.86	16.27 15.20	17.38 14.19	18.58 14.19	18.58 14.19	17.77 14.45	16.58 13.51	17.77 13.51	16.87 14.69	18.03 13.68	
0.8	0.57	15.67 14.60	16.69 13.68	15.98 14.92	17.01 13.94	18.16 13.94	15.24 14.18	16.28 13.23	17.29 13.23	15.46 14.39	16.50 13.47	17.54 13.47	15.20 14.19	16.22 13.31	17.38 13.31	15.48 14.46	16.55 13.54	17.70 13.54	15.81 13.81	16.92 12.92	16.04 12.92	17.15 13.12	15.02 14.00	17.15 13.12	
1	0.54	17.89 16.73	19.09 15.67	18.28 17.09	19.48 15.98	20.44 15.98	17.45 16.23	18.61 15.27	19.90 15.27	17.69 15.49	18.90 15.49	20.23 15.49	17.38 16.47	18.58 15.49	19.88 15.49	17.75 15.16	18.95 15.16	20.25 15.16	18.11 15.81	19.39 14.75	17.19 14.75	18.40 16.04	19.69 14.97		

^aFor the open-bubble inflation model accounting only for fluctuations generated during the evolution inside the bubble (RP94). At each Ω_0 , the first of the five entries in each of columns (3)–(10) is the maximum likelihood value, the first (vertical) pair delimits the 68.3% (1- σ) highest posterior density range, and the second (vertical) pair delimits the 95.5% (2- σ) highest posterior density range. ^bAccounting for (Yes), or ignoring (No) the correction for faint high-latitude foreground Galactic emission.

TABLE 3
 $Q_{\text{rms-PS}}$ Values for the $t_0 \simeq 13.5$ Gyr, $\Omega_B h^2 = 0.0205$ Open-Bubble Inflation Models^a

Frame:		Ecliptic					Galactic				
G.C. ^b :		No	No	Yes	Yes	No	No	Yes	Yes		
ℓ_{min} :		2	3	2	3	2	3	2	3		
Ω_0	h	$Q_{\text{rms-PS}}$ (μK)									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		
0.1	0.65	21.08 19.56 18.17	21.56 20.07 18.62	20.40 18.91 17.62	20.89 19.35 17.98	20.44 18.91 17.52	20.90 19.37 17.98	19.77 18.33 16.96	20.25 18.72 17.33		
0.2	0.61	24.23 22.52 20.90	24.84 23.05 21.44	23.53 21.86 20.18	24.04 22.34 20.65	23.44 21.78 20.20	24.05 22.33 20.71	22.76 21.09 19.52	23.26 21.59 19.97		
0.25	0.60	24.23 22.52 20.90	24.80 23.05 21.44	23.53 21.86 20.22	24.04 22.34 20.70	23.44 21.78 20.20	24.00 22.33 20.71	22.76 21.09 19.56	23.26 21.59 20.02		
0.3	0.59	23.54 21.92 20.39	24.12 22.41 20.88	22.76 21.25 19.81	23.27 21.69 20.21	22.84 21.22 19.69	23.35 21.73 20.20	22.16 20.54 19.09	22.61 20.99 19.51		
0.35	0.57	22.52 20.94 19.51	23.01 21.44 19.94	21.89 20.30 18.97	22.29 20.70 19.34	21.82 20.30 18.86	22.33 20.76 19.32	21.18 19.69 18.28	21.59 20.06 18.68		
0.4	0.56	21.22 19.79 18.44	21.69 20.20 18.88	20.51 19.23 17.88	20.96 19.56 18.25	20.62 19.19 17.84	21.04 19.60 18.26	19.99 18.62 17.30	20.39 18.95 17.66		
0.45	0.55	19.83 18.54 17.29	20.29 18.92 17.64	19.26 17.96 16.81	19.58 18.30 17.11	19.28 17.98 16.73	19.69 18.35 17.10	18.75 17.43 16.24	19.05 17.75 16.55		
0.5	0.55	18.54 17.29 16.13	18.92 17.64 16.49	18.01 16.81 15.69	18.34 17.11 15.95	17.98 16.78 15.62	18.35 17.15 15.99	17.47 16.28 15.17	17.80 16.59 15.44		
0.6	0.53	16.36 15.25 14.28	16.66 15.56 14.54	15.92 14.78 13.96	16.14 14.99 14.11	15.90 14.83 13.81	16.22 15.11 14.09	15.43 14.41 13.47	15.67 14.60 13.63		
0.8	0.50	15.57 14.56 13.63	15.86 14.83 13.90	15.21 14.19 13.24	15.39 14.32 13.42	15.11 14.14 13.21	15.39 14.42 13.44	14.75 13.77 12.87	14.93 13.91 13.03		
1	0.48	17.80 16.64 15.53	18.16 16.96 15.86	17.30 16.12 15.00	17.59 16.37 15.25	17.29 16.18 15.11	17.61 16.45 15.39	16.84 15.73 14.66	17.10 15.94 14.88		

^aFor the open-bubble inflation model accounting only for fluctuations generated during the evolution inside the bubble (RP94). At each Ω_0 , the first of the five entries in each of columns (3)–(10) is the maximum likelihood value, the first (vertical) pair delimits the 68.3% ($1-\sigma$) highest posterior density range, and the second (vertical) pair delimits the 95.5% ($2-\sigma$) highest posterior density range. ^bAccounting for (Yes), or ignoring (No) the correction for faint high-latitude foreground Galactic emission.

TABLE 4
 $Q_{\text{rms-PS}}$ Values for the $t_0 \simeq 10.5$ Gyr, $\Omega_B h^2 = 0.0055$ Flat-Space Scale-Invariant Spectrum Open Models^a

Frame:		Ecliptic					Galactic					
G.C. ^b :		No	No	Yes	Yes	No	No	Yes	Yes	Yes	Yes	
ℓ_{min} :		2	3	2	3	2	3	2	3	2	3	
Ω_0	h	$Q_{\text{rms-PS}}$ (μK)										
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)			
0.1	0.84	13.73 12.78	14.62 11.96	14.08 13.11	14.99 12.25	13.29 12.40	14.24 11.58	15.18 11.82	13.60 12.64	14.52 11.52	15.52 12.57	14.36 11.77
0.2	0.79	17.14 15.98	18.35 14.88	17.57 16.39	19.67 15.26	16.69 15.52	17.83 14.41	19.13 14.75	17.02 15.87	18.25 14.50	19.55 14.50	17.92 15.98
0.25	0.77	18.17 16.90	19.40 15.72	18.59 17.30	19.88 16.12	17.68 16.35	18.82 15.17	20.14 15.53	18.02 16.72	19.26 15.53	20.63 15.34	18.93 16.49
0.3	0.75	18.67 17.43	20.00 16.22	19.12 17.84	21.37 16.60	18.10 16.92	19.47 15.72	20.79 16.05	18.51 17.26	19.88 16.05	21.22 15.81	19.46 17.36
0.35	0.74	18.83 17.54	20.05 16.37	19.23 17.94	21.42 16.76	18.31 17.03	19.47 15.87	20.88 16.19	18.66 17.35	19.88 16.19	21.27 15.85	19.56 17.06
0.4	0.73	18.55 17.27	19.78 16.12	18.96 17.67	21.12 16.49	18.01 16.74	19.23 15.60	20.51 15.90	18.38 17.07	19.59 15.90	20.95 15.68	19.30 16.83
0.45	0.71	17.93 16.76	19.14 15.68	18.32 17.14	20.43 16.01	17.47 16.29	18.65 15.25	19.92 15.50	17.74 16.59	18.98 15.50	20.25 15.23	18.66 16.31
0.5	0.70	17.12 15.97	18.18 14.93	17.46 16.33	19.42 15.26	16.71 15.46	17.59 14.53	18.95 14.75	16.96 15.72	17.93 14.75	19.28 14.75	17.79 15.58
0.6	0.68	15.01 14.06	15.94 13.15	15.31 14.34	17.01 13.43	16.28 13.61	17.35 12.74	16.54 12.93	14.63 13.84	15.46 12.93	16.82 12.83	15.63 13.73
0.8	0.65	12.53 11.76	13.33 11.03	12.79 11.98	14.23 11.23	12.22 11.47	12.99 10.73	13.92 10.86	12.38 11.60	13.18 10.86	14.11 10.78	13.07 11.48
1	0.62	18.09 16.90	19.29 15.76	18.43 17.24	20.57 16.12	17.70 16.50	18.90 15.26	20.00 15.51	17.92 16.72	19.13 15.51	20.33 15.29	18.65 16.32

^aFor the flat-space scale-invariant spectrum open model (W83). The tabulated $Q_{\text{rms-PS}}$ values are determined from the conditional likelihood function at fixed Ω_0 . At each Ω_0 , the first of the five entries in each of columns (3)–(10) is the maximum likelihood value, the first (vertical) pair delimits the 68.3% (1- σ) highest posterior density range, and the second (vertical) pair delimits the 95.5% (2- σ) highest posterior density range. ^bAccounting for (Yes), or ignoring (No) the correction for faint high-latitude foreground Galactic emission.

TABLE 5
 $Q_{\text{rms-PS}}$ Values for the $t_0 \simeq 12$ Gyr, $\Omega_B h^2 = 0.0125$ Flat-Space Scale-Invariant Spectrum Open Models^a

Frame:		Ecliptic					Galactic				
G.C. ^b :		No	No	Yes	Yes	No	No	Yes	Yes		
ℓ_{min} :		2	3	2	3	2	3	2	3		
Ω_0	h	$Q_{\text{rms-PS}}$ (μK)									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		
0.1	0.73	13.67 12.79 11.95	14.02 13.11 12.25	13.24 12.37 11.59	13.54 12.66 11.82	13.46 12.59 11.76	13.79 12.90 12.06	13.06 12.20 11.39	13.33 12.47 11.63		
0.2	0.69	17.14 15.98 14.88	17.57 16.39 15.26	16.71 15.52 14.40	17.02 15.85 14.73	16.70 15.57 14.49	17.16 15.98 14.85	16.22 15.10 14.02	16.57 15.44 14.34		
0.25	0.67	18.13 16.83 15.74	18.59 17.24 16.12	17.63 16.34 15.21	17.99 16.68 15.57	17.64 16.40 15.33	18.09 16.82 15.70	17.14 15.90 14.83	17.50 16.25 15.18		
0.3	0.66	18.61 17.37 16.16	19.07 17.78 16.55	18.00 16.88 15.68	18.43 17.23 16.00	18.17 16.89 15.76	18.57 17.32 16.16	17.62 16.39 15.26	18.00 16.75 15.59		
0.35	0.65	18.72 17.48 16.26	19.12 17.89 16.65	18.20 16.95 15.72	18.53 17.30 16.07	18.21 17.00 15.87	18.64 17.39 16.24	17.67 16.49 15.36	18.02 16.83 15.70		
0.4	0.63	18.45 17.18 16.08	18.86 17.57 16.44	17.89 16.62 15.57	18.24 16.96 15.87	18.00 16.73 15.64	18.38 17.10 15.99	17.46 16.22 15.15	17.79 16.53 15.44		
0.45	0.62	17.83 16.69 15.55	18.21 17.03 15.90	17.35 16.23 15.07	17.66 16.51 15.35	17.40 16.22 15.17	17.77 16.57 15.50	16.92 15.74 14.72	17.23 16.03 14.98		
0.5	0.61	17.00 15.88 14.89	17.35 16.22 15.20	16.57 15.40 14.44	16.82 15.67 14.67	16.57 15.48 14.51	16.93 15.80 14.81	16.12 15.04 14.08	16.38 15.29 14.30		
0.6	0.60	14.92 13.97 13.06	15.20 14.24 13.32	14.56 13.62 12.72	14.77 13.81 12.87	14.58 13.65 12.75	14.89 13.94 13.03	14.18 13.27 12.41	14.41 13.48 12.56		
0.8	0.57	12.48 11.64 10.91	12.73 11.87 11.12	12.17 11.33 10.58	12.33 11.47 10.73	12.17 11.43 10.67	12.41 11.65 10.85	11.87 11.12 10.37	12.02 11.26 10.50		
1	0.54	17.87 16.72 15.64	18.26 17.08 15.96	17.43 16.22 15.24	17.67 16.47 15.46	17.34 16.25 15.13	17.71 16.57 15.45	16.92 15.79 14.72	17.15 16.02 14.94		

^aFor the flat-space scale-invariant spectrum open model (W83). At each Ω_0 , the first of the five entries in each of columns (3)–(10) is the maximum likelihood value, the first (vertical) pair delimits the 68.3% (1- σ) highest posterior density range, and the second (vertical) pair delimits the 95.5% (2- σ) highest posterior density range. ^bAccounting for (Yes), or ignoring (No) the correction for faint high-latitude foreground Galactic emission.

TABLE 6
 $Q_{\text{rms-PS}}$ Values for the $t_0 \simeq 13.5$ Gyr, $\Omega_B h^2 = 0.0205$ Flat-Space Scale-Invariant Spectrum Open Models^a

Frame:		Ecliptic					Galactic				
G.C. ^b :		No	No	Yes	Yes	No	No	Yes	Yes		
ℓ_{min} :		2	3	2	3	2	3	2	3		
Ω_0	h	$Q_{\text{rms-PS}}$ (μK)									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		
0.1	0.65	13.70 12.79 11.95	14.02 13.11 12.25	13.28 12.39 11.57	13.58 12.66 11.81	13.47 12.53 11.77	13.79 12.85 12.08	13.06 12.14 11.39	13.35 12.41 11.63		
0.2	0.61	17.09 15.93 14.83	17.51 16.33 15.20	16.60 15.43 14.34	16.96 15.79 14.66	16.72 15.53 14.50	17.15 15.93 14.87	16.22 15.04 14.02	16.59 15.40 14.34		
0.25	0.60	18.09 16.83 15.66	18.53 17.24 16.06	17.59 16.34 15.16	17.95 16.69 15.50	17.58 16.40 15.26	18.02 16.82 15.64	17.08 15.90 14.77	17.45 16.26 15.10		
0.3	0.59	18.56 17.32 16.17	19.02 17.73 16.55	17.94 16.79 15.70	18.37 17.15 16.01	18.11 16.86 15.70	18.51 17.26 16.09	17.57 16.33 15.20	17.94 16.69 15.54		
0.35	0.57	18.64 17.37 16.24	19.07 17.78 16.60	18.14 16.84 15.79	18.48 17.17 16.07	18.14 16.95 15.79	18.58 17.33 16.18	17.62 16.44 15.31	17.96 16.75 15.62		
0.4	0.56	18.34 17.13 16.01	18.75 17.51 16.39	17.78 16.65 15.51	18.15 16.96 15.82	17.88 16.71 15.57	18.26 17.10 15.93	17.35 16.22 15.10	17.70 16.53 15.38		
0.45	0.55	17.75 16.63 15.53	18.16 16.98 15.85	17.24 16.12 15.10	17.55 16.41 15.35	17.29 16.18 15.11	17.66 16.52 15.45	16.81 15.69 14.67	17.10 15.97 14.92		
0.5	0.55	16.89 15.84 14.84	17.24 16.17 15.15	16.44 15.42 14.43	16.73 15.68 14.65	16.53 15.43 14.44	16.89 15.77 14.76	16.06 14.99 14.02	16.36 15.26 14.24		
0.6	0.53	14.86 13.91 13.04	15.15 14.18 13.27	14.45 13.48 12.68	14.67 13.68 12.85	14.54 13.61 12.72	14.84 13.86 12.95	14.13 13.22 12.36	14.35 13.40 12.52		
0.8	0.50	12.39 11.60 10.86	12.63 11.82 11.07	12.10 11.31 10.58	12.24 11.44 10.70	12.11 11.37 10.64	12.35 11.59 10.85	11.82 11.07 10.37	11.96 11.20 10.50		
1	0.48	17.80 16.65 15.52	18.16 16.98 15.85	17.30 16.13 14.99	17.59 16.38 15.25	17.27 16.13 15.11	17.59 16.41 15.39	16.81 15.69 14.67	17.08 15.90 14.88		

^aFor the flat-space scale-invariant spectrum open model (W83). At each Ω_0 , the first of the five entries in each of columns (3)–(10) is the maximum likelihood value, the first (vertical) pair delimits the 68.3% (1- σ) highest posterior density range, and the second (vertical) pair delimits the 95.5% (2- σ) highest posterior density range. ^bAccounting for (Yes), or ignoring (No) the correction for faint high-latitude foreground Galactic emission.

TABLE 7
 $Q_{\text{rms-PS}}$ Values for the $h = 0.6$, $\Omega_B = 0.035$ Open-Bubble Inflation Models^a

Frame:		Ecliptic					Galactic																		
G.C. ^b :	ℓ_{min} :	No	No	Yes	Yes	No	No	Yes	Yes																
Ω_0	t_0 (Gyr)	$Q_{\text{rms-PS}}$ (μK)																							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)																
0.1	14.6	26.45 24.56	28.49 22.75	30.62 25.20	27.15 23.40	29.19 23.77	31.41 21.95	25.71 24.37	27.71 22.50	29.74 22.50	26.27 24.37	28.32 22.56	30.48 21.22	25.57 24.37	27.56 22.56	29.69 21.22	26.27 24.37	28.31 22.94	30.48 21.22	24.83 22.94	26.73 22.94	28.81 21.22	25.39 23.54	27.38 21.73	29.56 21.73
0.2	13.8	25.39 23.58	27.29 21.92	29.32 21.32	26.04 24.19	27.94 22.47	30.06 21.32	24.59 22.78	26.50 21.32	28.54 21.75	25.19 23.33	27.10 21.75	29.19 21.75	24.60 22.84	26.45 21.13	28.44 21.13	27.10 23.40	29.19 21.73	23.86 22.10	25.67 20.48	27.66 20.48	24.42 22.61	26.27 22.61	28.31 20.94	23.31 20.94
0.3	13.2	23.40 21.78	25.11 20.25	26.92 20.76	23.95 22.29	25.67 20.76	27.56 21.13	22.64 19.54	24.37 21.55	26.22 20.00	23.15 21.55	24.88 21.55	26.78 20.00	22.70 21.08	24.37 19.60	26.13 20.06	23.21 21.59	24.93 20.06	26.78 18.95	22.01 20.44	23.63 18.95	25.44 20.44	22.47 20.85	24.14 19.37	25.99 19.37
0.4	12.7	21.13 19.74	22.66 18.40	24.28 18.40	21.64 20.16	23.17 18.77	24.79 18.77	20.47 19.24	22.11 17.90	23.63 17.90	20.93 19.57	22.53 18.19	24.09 18.19	20.53 19.09	21.96 17.80	23.58 17.80	20.99 19.56	22.40 18.21	21.93 18.54	22.36 17.24	22.94 17.24	20.34 18.91	21.82 17.56	23.40 17.56	
0.5	12.3	19.09 17.80	20.39 16.55	21.78 16.92	19.46 18.17	20.81 16.92	22.24 16.92	18.64 17.29	19.83 16.08	21.27 16.08	18.93 17.61	20.20 16.31	21.59 16.31	18.49 17.24	19.79 16.08	21.18 16.08	18.91 17.61	20.20 16.45	21.64 16.45	17.98 16.73	19.23 15.62	20.67 15.62	18.31 17.06	19.60 15.85	20.99 15.85
0.6	11.9	17.33 16.18	18.54 15.20	19.83 15.20	17.75 16.55	18.95 15.44	20.20 15.44	16.64 15.65	18.02 14.87	19.38 14.87	17.06 15.91	18.33 15.04	19.67 15.04	16.96 15.81	18.07 14.69	19.32 14.69	17.24 16.13	18.44 15.02	19.74 14.23	16.50 15.34	17.61 14.23	18.81 14.23	16.78 15.57	17.89 14.46	19.14 14.46
0.7	11.6	16.36 15.34	17.47 14.46	18.72 14.46	16.69 15.67	17.84 14.74	19.09 14.74	15.90 14.88	17.01 14.09	18.26 14.28	16.13 15.11	17.29 14.28	18.54 14.28	16.04 14.93	17.10 13.91	18.26 13.91	16.36 15.25	17.47 14.19	18.63 14.19	15.57 14.46	16.64 14.46	17.80 13.54	15.81 14.69	16.92 13.72	18.07 13.72
0.8	11.3	16.27 15.30	17.38 14.42	18.63 14.42	16.55 15.57	17.70 14.65	18.95 14.65	15.92 14.90	16.87 14.11	18.22 14.26	16.13 15.10	17.15 14.26	18.47 14.26	15.90 14.83	17.01 13.81	18.12 13.81	16.27 15.16	17.33 14.09	18.49 14.09	15.44 14.37	16.50 13.44	17.66 13.44	15.71 14.60	16.78 13.63	17.94 13.63
0.9	11.1	16.82 15.76	17.98 14.83	19.23 14.83	17.19 16.08	18.35 15.11	19.65 15.11	16.30 15.30	17.45 14.42	18.71 14.60	16.61 15.53	17.72 14.60	19.02 14.60	16.50 15.39	17.61 14.32	18.77 14.32	16.82 15.71	17.94 14.60	19.14 14.60	16.04 14.93	17.15 13.91	18.31 13.91	16.31 15.16	17.38 14.09	18.58 14.09

^aFor the open-bubble inflation model accounting for fluctuations generated during the evolution inside the bubble (RP94), as well as those that come into the bubble from the first epoch of inflation (BGT; YST), and those that correspond to a non-square-integrable basis function (YST). At each Ω_0 , the first of the five entries in each of columns (3)–(10) is the maximum likelihood value, the first (vertical) pair delimits the 68.3% (1σ) highest posterior density range, and the second (vertical) pair delimits the 95.5% (2σ) highest posterior density range. ^bAccounting for (Yes), or ignoring (No) the correction for faint high-latitude foreground Galactic emission.

TABLE 8
Gauge-Invariant Fractional Energy-Density Perturbation Power Spectrum Normalization Factor Ah^4 ^a

Model:	Inflation ^b	Inflation ^c	Scale-Inv. ^d
Ω_0	Ah^4 (10^5Mpc^4)	Ah^4 (10^5Mpc^4)	Ah^4 (10^5Mpc^4)
(1)	(2)	(3)	(4)
0.1	1.49	1.42	7.00
0.2	1.36	1.34	3.94
0.25	1.44	1.43	3.52
0.3	1.58	1.58	3.34
0.35	1.78	1.78	3.31
0.4	2.03	2.03	3.40
0.45	2.34	2.34	3.60
0.5	2.69	2.68	3.89
0.6	3.37	3.35	4.80
0.8	3.26	3.24	5.59
1	2.03	2.03	2.03

^aNormalized to $Q_{\text{rms-PS}} = 10 \mu\text{K}$, and scale like $(Q_{\text{rms-PS}}/10 \mu\text{K})^2$. These are computed for $t_0 \simeq 12 \text{ Gyr}$ and $\Omega_B h^2 = 0.0125$, and over the range of these parameters considered here only the third significant figure in the numerical value for A depends (weakly) on the values of these parameters. ^bFor the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94).

^cFor the open-bubble inflation model, now also accounting for fluctuations generated in the first epoch of inflation (BGT; YST). ^dFor the flat-space scale-invariant spectrum open model (W83).

TABLE 9
Numerical Values for the $t_0 \simeq 10.5 \text{ Gyr}$, $\Omega_B h^2 = 0.0055$ Models

Model:				Inflation ^a			Scale-Inv. ^b		
Ω_0	h	$\frac{\Omega_B}{\Omega_0}$	S	$Q_{\text{rms-PS}}$ (μK)	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}$	β_I	$Q_{\text{rms-PS}}$ (μK)	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}$	β_I
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.1	0.84	0.078	0.075	16.98 – 24.96	0.105 – 0.154	0.034 – 0.050	11.39 – 16.01	0.152 – 0.213	0.050 – 0.070
0.2	0.79	0.044	0.15	19.56 – 28.64	0.282 – 0.413	0.14 – 0.20	14.02 – 20.14	0.343 – 0.493	0.17 – 0.24
0.25	0.77	0.037	0.18	19.62 – 28.58	0.379 – 0.552	0.21 – 0.31	14.83 – 21.27	0.446 – 0.641	0.25 – 0.36
0.3	0.75	0.033	0.21	19.18 – 27.82	0.474 – 0.688	0.30 – 0.43	15.31 – 21.86	0.549 – 0.784	0.35 – 0.49
0.35	0.74	0.029	0.25	18.40 – 26.56	0.575 – 0.830	0.40 – 0.57	15.36 – 21.92	0.655 – 0.934	0.45 – 0.65
0.4	0.73	0.026	0.28	17.42 – 25.04	0.674 – 0.968	0.51 – 0.73	15.20 – 21.59	0.760 – 1.08	0.57 – 0.81
0.45	0.71	0.024	0.31	16.38 – 23.40	0.756 – 1.08	0.61 – 0.87	14.77 – 20.84	0.845 – 1.19	0.68 – 0.96
0.5	0.70	0.022	0.34	15.32 – 21.80	0.843 – 1.20	0.72 – 1.0	14.13 – 19.82	0.936 – 1.31	0.80 – 1.1
0.6	0.68	0.020	0.39	13.56 – 19.22	0.996 – 1.41	0.95 – 1.4	12.46 – 17.35	1.09 – 1.52	1.0 – 1.5
0.8	0.65	0.016	0.50	13.04 – 18.32	1.22 – 1.72	1.4 – 2.0	10.48 – 14.51	1.29 – 1.78	1.5 – 2.0
1	0.62	0.014	0.60	14.86 – 21.02	1.31 – 1.86	1.7 – 2.4	14.83 – 21.00	1.31 – 1.85	1.7 – 2.4

^aFor the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94). Two standard deviation range. ^bFor the flat-space scale-invariant spectrum open model (W83). Two standard deviation range.

TABLE 10
Numerical Values for the $t_0 \simeq 12$ Gyr, $\Omega_B h^2 = 0.0125$ Models

Model:				Inflation ^a				Scale-Inv. ^b			
Ω_0	h	$\frac{\Omega_B}{\Omega_0}$	S	$Q_{\text{rms-PS}}$	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}$	β_I	$Q_{\text{rms-PS}}$	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}$	β_I		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		
0.1	0.73	0.23	0.054	16.98 – 24.98	0.0663 – 0.0976	0.022 – 0.032	11.39 – 16.01	0.0965 – 0.136	0.032 – 0.044		
0.2	0.69	0.13	0.12	19.56 – 28.64	0.204 – 0.298	0.10 – 0.15	14.02 – 20.14	0.248 – 0.357	0.12 – 0.18		
0.25	0.67	0.11	0.14	19.60 – 28.54	0.279 – 0.406	0.16 – 0.23	14.83 – 21.27	0.330 – 0.473	0.19 – 0.27		
0.3	0.66	0.096	0.17	19.14 – 27.76	0.362 – 0.525	0.23 – 0.33	15.26 – 21.81	0.419 – 0.599	0.26 – 0.38		
0.35	0.65	0.085	0.20	18.36 – 26.48	0.445 – 0.642	0.31 – 0.44	15.36 – 21.86	0.508 – 0.723	0.35 – 0.50		
0.4	0.63	0.079	0.22	17.38 – 24.94	0.514 – 0.738	0.39 – 0.55	15.15 – 21.49	0.580 – 0.823	0.44 – 0.62		
0.45	0.62	0.072	0.25	16.32 – 23.28	0.591 – 0.844	0.48 – 0.68	14.72 – 20.74	0.661 – 0.931	0.53 – 0.75		
0.5	0.61	0.067	0.27	15.26 – 21.68	0.663 – 0.942	0.57 – 0.81	14.08 – 19.71	0.736 – 1.03	0.63 – 0.88		
0.6	0.60	0.058	0.33	13.50 – 19.10	0.807 – 1.14	0.77 – 1.1	12.41 – 17.24	0.884 – 1.23	0.85 – 1.2		
0.8	0.57	0.048	0.42	12.98 – 18.20	1.00 – 1.41	1.1 – 1.6	10.37 – 14.40	1.05 – 1.46	1.2 – 1.7		
1	0.54	0.043	0.49	14.74 – 20.84	1.08 – 1.52	1.4 – 2.0	14.72 – 20.79	1.08 – 1.52	1.4 – 2.0		

^aFor the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94). Two standard deviation range. ^bFor the flat-space scale-invariant spectrum open model (W83). Two standard deviation range.

TABLE 11
Numerical Values for the $t_0 \simeq 13.5$ Gyr, $\Omega_B h^2 = 0.0205$ Models

Model:				Inflation ^a				Scale-Inv. ^b			
Ω_0	h	$\frac{\Omega_B}{\Omega_0}$	S	$Q_{\text{rms-PS}}$	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}$	β_I	$Q_{\text{rms-PS}}$	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}$	β_I		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		
0.1	0.65	0.49	0.036	17.00 – 24.98	0.0353 – 0.0519	0.012 – 0.017	11.39 – 16.01	0.0515 – 0.0724	0.017 – 0.024		
0.2	0.61	0.28	0.085	19.56 – 28.60	0.135 – 0.197	0.067 – 0.098	14.02 – 20.09	0.165 – 0.236	0.082 – 0.12		
0.25	0.60	0.23	0.11	19.58 – 28.50	0.198 – 0.288	0.11 – 0.16	14.77 – 21.22	0.233 – 0.335	0.13 – 0.19		
0.3	0.59	0.20	0.13	19.12 – 27.70	0.264 – 0.382	0.17 – 0.24	15.20 – 21.76	0.305 – 0.436	0.19 – 0.28		
0.35	0.57	0.18	0.15	18.32 – 26.40	0.321 – 0.462	0.22 – 0.32	15.31 – 21.76	0.366 – 0.520	0.25 – 0.36		
0.4	0.56	0.16	0.18	17.32 – 24.86	0.385 – 0.552	0.29 – 0.41	15.10 – 21.38	0.434 – 0.615	0.33 – 0.46		
0.45	0.55	0.15	0.20	16.28 – 23.20	0.447 – 0.638	0.36 – 0.51	14.67 – 20.63	0.500 – 0.703	0.40 – 0.57		
0.5	0.55	0.14	0.22	15.20 – 21.60	0.520 – 0.740	0.45 – 0.63	14.02 – 19.61	0.578 – 0.808	0.50 – 0.69		
0.6	0.53	0.12	0.26	13.46 – 19.02	0.626 – 0.885	0.60 – 0.85	12.36 – 17.14	0.686 – 0.951	0.66 – 0.91		
0.8	0.50	0.10	0.33	12.92 – 18.08	0.790 – 1.11	0.90 – 1.3	10.37 – 14.29	0.830 – 1.14	0.94 – 1.3		
1	0.48	0.089	0.40	14.64 – 20.68	0.878 – 1.24	1.1 – 1.6	14.67 – 20.63	0.880 – 1.24	1.1 – 1.6		

^aFor the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94). Two standard deviation range. ^bFor the flat-space scale-invariant spectrum open model (W83). Two standard deviation range.

TABLE 12
Numerical Values for the $t_0 \simeq 12$ Gyr, $\Omega_B h^2 = 0.007$ Models^a

Model:				Inflation ^b				Scale-Inv. ^c			
Ω_0	h	$\frac{\Omega_B}{\Omega_0}$	S	$Q_{\text{rms-PS}}$	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}$	β_I	$Q_{\text{rms-PS}}$	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}$	β_I		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		
0.1	0.73	0.13	0.061	16.98 – 24.96	0.0795 – 0.117	0.026 – 0.038	11.39 – 16.01	0.116 – 0.163	0.038 – 0.053		
0.2	0.69	0.074	0.12	19.56 – 28.64	0.226 – 0.331	0.11 – 0.16	14.02 – 20.14	0.275 – 0.396	0.14 – 0.20		
0.25	0.67	0.062	0.15	19.62 – 28.58	0.305 – 0.444	0.17 – 0.25	14.83 – 21.27	0.360 – 0.517	0.20 – 0.29		
0.3	0.66	0.054	0.18	19.18 – 27.82	0.391 – 0.568	0.25 – 0.36	15.31 – 21.86	0.454 – 0.648	0.29 – 0.41		
0.35	0.65	0.047	0.21	18.40 – 26.56	0.477 – 0.689	0.33 – 0.48	15.36 – 21.92	0.543 – 0.775	0.38 – 0.54		
0.4	0.63	0.044	0.24	17.42 – 25.04	0.549 – 0.789	0.41 – 0.59	15.20 – 21.59	0.620 – 0.881	0.47 – 0.66		
0.45	0.62	0.040	0.26	16.38 – 23.40	0.629 – 0.898	0.51 – 0.72	14.77 – 20.84	0.703 – 0.992	0.57 – 0.80		
0.5	0.61	0.038	0.29	15.32 – 21.80	0.703 – 1.00	0.60 – 0.86	14.13 – 19.82	0.780 – 1.09	0.67 – 0.94		
0.6	0.60	0.032	0.34	13.56 – 19.22	0.849 – 1.20	0.81 – 1.2	12.46 – 17.35	0.930 – 1.30	0.89 – 1.2		
0.8	0.57	0.027	0.43	13.04 – 18.32	1.05 – 1.47	1.2 – 1.7	10.48 – 14.51	1.10 – 1.53	1.3 – 1.7		
1	0.54	0.024	0.51	14.86 – 21.02	1.13 – 1.59	1.5 – 2.1	14.83 – 21.00	1.12 – 1.59	1.5 – 2.1		

^aNormalized to $Q_{\text{rms-PS}}$ for the $t_0 \simeq 10.5$ Gyr, $\Omega_B h^2 = 0.0055$ models. ^bFor the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94). Two standard deviation range. ^cFor the flat-space scale-invariant spectrum open model (W83). Two standard deviation range.

TABLE 13
Numerical Values for the $h = 0.6$, $\Omega_B = 0.035$ Open-Bubble Inflation Models

Ω_0	$Q_{\text{rms-PS}}^a$	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}^a$	$Q_{\text{rms-PS}}^b$	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}^b$	$Q_{\text{rms-PS}}^c$	$\frac{\delta M}{M} _{8h^{-1}\text{Mpc}}^c$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.1	16.96 – 24.93	0.0422 – 0.0620	17.24 – 25.34	0.0422 – 0.0620	21.22 – 31.41	0.0412 – 0.0610
0.2	19.56 – 28.68	0.159 – 0.233	19.65 – 28.81	0.158 – 0.232	20.48 – 30.06	0.156 – 0.230
0.3	19.14 – 27.75	0.311 – 0.450	19.14 – 27.75	0.310 – 0.450	18.95 – 27.56	0.306 – 0.445
0.4	17.38 – 24.88	0.478 – 0.684	17.33 – 24.88	0.476 – 0.684	17.24 – 24.79	0.471 – 0.678
0.5	15.25 – 21.69	0.647 – 0.921	15.25 – 21.69	0.647 – 0.920	15.62 – 22.24	0.642 – 0.914
0.6	13.49 – 19.05	0.805 – 1.14	13.54 – 19.09	0.805 – 1.14	14.23 – 20.20	0.796 – 1.13
0.7	12.80 – 17.89	0.953 – 1.33	12.84 – 17.94	0.952 – 1.33	13.54 – 19.09	0.940 – 1.33
0.8	12.98 – 18.17	1.06 – 1.49	12.98 – 18.21	1.06 – 1.49	13.44 – 18.95	1.06 – 1.49
0.9	13.72 – 19.37	1.15 – 1.62	13.72 – 19.37	1.14 – 1.62	13.91 – 19.65	1.15 – 1.62

^aFor the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94). Two standard deviation range. ^bFor the open-bubble inflation model, now also accounting for fluctuations generated in the first epoch of inflation (BGT; YST). Two standard deviation range. ^cFor the open-bubble inflation model, now also accounting for both the fluctuations generated in the first epoch of inflation as well those that correspond to a non-square-integrable basis function (YST). Two standard deviation range.

TABLE 14
Approximate Fitting Formulae for $(\delta M/M)[8h^{-1} \text{ Mpc}]^a$

Model:			Inflation ^b	Scale-Inv. ^c
t_0 (Gyr)	$\Omega_B h^2$	Limit	$\frac{\delta M}{M} _{8h^{-1} \text{ Mpc}}$	$\frac{\delta M}{M} _{8h^{-1} \text{ Mpc}}$
(1)	(2)	(3)	(4)	(5)
$\simeq 10.5$	0.0055	+2- σ	$1.92 \sin[0.5\pi(\Omega_0 - 0.065)]$	$1.88 \sin[0.55\pi(\Omega_0 - 0.05)]$
$\simeq 10.5$	0.0055	-2- σ	$1.34 \sin[0.5\pi(\Omega_0 - 0.065)]$	$1.31 \sin[0.55\pi(\Omega_0 - 0.05)]$
$\simeq 12$	0.0125	+2- σ	$1.52 \sin[0.5\pi(\Omega_0 - 0.075)]$	$1.52 \sin[0.55\pi(\Omega_0 - 0.065)]$
$\simeq 12$	0.0125	-2- σ	$1.08 \sin[0.5\pi(\Omega_0 - 0.080)]$	$1.08 \sin[0.55\pi(\Omega_0 - 0.065)]$
$\simeq 13.5$	0.0205	+2- σ	$1.23 \sin[0.495\pi(\Omega_0 - 0.095)]$	$1.25 \sin[0.525\pi(\Omega_0 - 0.085)]$
$\simeq 13.5$	0.0205	-2- σ	$0.87 \sin[0.495\pi(\Omega_0 - 0.100)]$	$0.89 \sin[0.525\pi(\Omega_0 - 0.085)]$
$\simeq 12$	0.007	+2- σ	$1.61 \sin[0.5\pi(\Omega_0 - 0.070)]$	$1.59 \sin[0.55\pi(\Omega_0 - 0.055)]$
$\simeq 12$	0.007	-2- σ	$1.14 \sin[0.5\pi(\Omega_0 - 0.075)]$	$1.13 \sin[0.55\pi(\Omega_0 - 0.060)]$

^aFor the $\pm 2\sigma$ entries of cols. (6) and (9) of Tables 9–12. The fits are good to better than $\sim 3\%$ for $0.2 \leq \Omega_0 \leq 1$. ^bFor the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94). ^cFor the flat-space scale-invariant spectrum open model (W83).

TABLE 15
Numerical Values for a “Central” $(\delta M/M)[8h^{-1} \text{ Mpc}]$ Value^a

Model:		Inflat. ^b				Sc.-Inv. ^c			
t_0 (Gyr) \simeq	$\Omega_B h^2 =$	10.5	12	13.5	12	10.5	12	13.5	12
Ω_0	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0.1		0.129	0.0820	0.0436	0.0982	0.183	0.116	0.0620	0.139
0.2		0.348	0.251	0.166	0.278	0.418	0.303	0.200	0.336
0.25		0.465	0.343	0.243	0.375	0.544	0.401	0.284	0.438
0.3		0.581	0.443	0.323	0.479	0.667	0.509	0.371	0.551
0.35		0.703	0.543	0.391	0.583	0.794	0.615	0.443	0.659
0.4		0.821	0.626	0.468	0.669	0.920	0.702	0.524	0.750
0.45		0.918	0.717	0.543	0.764	1.02	0.796	0.602	0.848
0.5		1.02	0.803	0.630	0.852	1.12	0.884	0.693	0.937
0.6		1.20	0.974	0.756	1.03	1.30	1.06	0.818	1.11
0.8		1.47	1.20	0.948	1.26	1.54	1.25	0.987	1.31
1		1.58	1.30	1.06	1.36	1.58	1.30	1.06	1.36

^aMean of the $\pm 2\sigma$ entries of cols. (6) and (9) of Tables 9–12. ^bFor the open-bubble inflation model, accounting only for fluctuations generated during the evolution inside the bubble (RP94). ^cFor the flat-space scale-invariant spectrum open model (W83).